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SUMMARY REPORT
ON
THE TEACHING OF MATHEMATICS
IN
JAPAN

By R. FUJISAWA

TOKIO
1912

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PREFACE.

Following the resolution passed at the fourth International Congress of Mathematicians held at Rome in April 1908, an International Commission was formed to make a comparative study of methods and ways in the teaching of mathematics in the secondary schools of different countries and present a collective report to the next Congress to be held at Cambridge in August 1912.

The words "secondary school" have varied meanings in different countries. This, in conjunction with other considerations regarding the working detail of the enquiry, soon lead to the conviction, that such limitation is not only undesirable but even impracticable. So it was altogether dropped. It was decided that the work of the Commission should include the whole field of mathematical instruction, from the most rudimentary education to that stage of the university education, where original research begins.

Preliminary report on the organization of the Commission and the general plan of its work, which will be hereafter referred to by the name of *Rapport Prdiminaire*, was published in November 1908, and very soon afterwards, national Sub-Commissions were formed and the work began energetically in most of the principal countries of the world.

It is much to be regretted that, in the case of Japan through various complex circumstances of more or less accidental nature, the invitation to join this movement could effectively brought to the official notice only so late as

towards the end of the summer of 1910. Not a little have we hesitated in accepting the invitation in view of the extreme limitation of time at our disposal. Various difficulties external to the sphere of the real work of the enquiry were also not to be lost sight of. We would have been very fortunate, if we could have begun the actual work before the close of 1910. From the start, it was evident that to have the report ready, translated into one of the official languages allowed, printed, and sent to the *Secrétariat-Général* early in 1911, as suggested in the *Rapport Préliminaire*, was altogether out of the question. However, the desire to participate in a world-wide movement of such magnitude and importance, proved itself to be a sufficient inducement for acceding to the wishes which reached us from various quarters, by accepting, though reluctantly, the invitation, notwithstanding that we were fully conscious of the difficulties which are most likely unique in the case of Japan under the present circumstances.

The invitation was accepted under the following condition: Shortness of time still remaining does not permit us, however desirable it might be, to send the printed report in one of the official languages to the *Sécrétariat-Général* before the first of August 1911, as is required in the case of other countries, but we shall try our best and have it ready for distribution some time before the Cambridge meeting of 1912.

Even under the stipulation just mentioned, the whole available time did not much exceed one year, a considerable portion of which must be assigned to the work of

translation into English which is the official language adopted. Indeed greater difficulty was actually experienced in this work of translation, than was anticipated at the time of starting.

In order not to lose even a little of this precious time, the Japanese translation of the *Rapport Prĕliminaire* prepared by myself, together with its French original and English and German translations, was published in October 1910. Copies of this pamphlet were sent to different schools, either directly, or indirectly through the authorities under whose control such schools stand, accompanied by the request to send in written reports on various points mentioned therein, so far as they touch the teaching of mathematics in the respective schools. No less than five hundred copies were thus distributed all over the country, and, in due course of time, no less than two hundred reports, which will be briefly designated as the "local reports," were received from the following schools:

Schools under the Department of Education

The Faculty of Agriculture of the Tōhoku Imperial University.
The Hiroshima Higher Normal School.
The Nara Higher Normal School for Women.
The Second, the Third, the Fourth, the Fifth, the Sixth, and the Seventh Higher Middle School.
The Technical Colleges of Ōsaka, Yonezawa, Sendai (since attached to the Tōhoku Imperial University), Nagoya, and Kumamoto.
The Commercial Colleges of Kobé, Nagasaki, and Yamaguchi.
The Agricultural Colleges of Morioka, and Kagoshima.
The Third Temporary Institute for the Training of Teachers.

Schools under the other Government Departments.

The Nobles' School [under the Imperial Household Department].
The Naval College.
The Naval Engineering College.
The Mercantile-Marine School [Shōsen-Gakkō].
The Training School for Officials in the Department of Communications.
The Military College.
The Military Artillery and Engineering College.
The (Military) Training School for Land Surveying.

Twenty-five Elementary Schools.

Fifty-four Middle Schools.

Twenty-six Girls' High Schools.

Thirty-one Normal Schools.

Five Normal Schools for Women.

Thirteen Agricultural Schools.

Nine Commercial Schools.

Seven Technical Schools.

The Awashima Prefectural Navigation School.

Besides these, we have received some more reports directly addressed to some of the members of our Sub-Commission, of which we shall speak presently.

In the mean time, was formed the Japanese Sub-Commission consisting of the members, whose names are appended to this preface. However desirable it might have been to ask for the co-operation of those who live elsewhere, it was thought necessary, in view of the utter insufficiency of time at our disposal, to confine our choise to those who reside on the spot, where the work of the Sub-Commission was being carried out. But with all our precaution, it was not possible

to avoid that some of the members had to leave in the midst of our work, in response to the call of the newly established Faculty of Science of the Tōhoku Imperial University at Sendai. In a few cases, this was anticipated; but they had to leave for the seat of the new University sooner than was expected, and on the other hand, our work had not made such a progress as to come up to our expectation that it would be finished at least in its main structure before they should have left Tokio.

The first meeting of our Sub-Commission took place at the Department of Education on the 14th of January 1911. The members were divided into a certain number of Divisions, each of which was entrusted with the task of dealing with a certain type of schools. The local reports mentioned above, were classified as soon as they were received, and distributed among the Divisions. Each Division was to prepare a report in Japanese, to be called Divisional Report, based partly upon the local reports and partly upon investigations and discussions of its own. And then these Divisional Reports were to be translated into English. Since then, our Sub-Commission held its general meeting twice, namely, on the 20th of May 1911 and on the 29th of October of the same year.

It was desired that the principal points of the reports be presented and discussed at educational gatherings and scientific societies which interest themselves in the progress of the teaching of mathematics, so that they might, when finally submitted, reflect the sentiment of the majority of those who are most competent to judge of such affairs. In this direction,

the lack of time was felt more severely than in any other direction. It was not even possible to discuss such points in a general meeting of our own. Again, in the subsequent work of translation, we have experienced, as was said before, difficulties far surpassing our most pessimistic forebodings. We had to content ourselves with the least evil.

However crude, and even unsatisfactory to ourselves, the results of our labour came out, we felt bound by our promise, and so, both the original and the English translation of the fifteen Divisional Reports, whose list is to be found after the member-list, have been published. The utter lack of time will account for very many shortcomings both in form and content to be found in them. Indeed, it is much to be regretted that our best intentions were forbidden the support of our best energies.

Besides these Divisional Reports was to be published the present summary report which should give a *resumé* and also contain matters not included in them. The task of preparing it was assigned to myself. To save time, I had to go through the awkward task of writing a summary of what I have never seen. About one-third of the present report was written and printed, when Divisional Reports began to come in intermittently. Now all of my time which I had allotted in my mind to what remained of my own work, was actually more than exhausted in trying to conquer, even partly, the aforesaid difficulties connected with the work of translation of the Divisional Reports. Thus but little time was left to me for doing my own work. All I could do was to write, as fast as I could, just what I happened

to know. So, not only where I was expressing my own personal views, but wherever there was the slightest doubt in this respect, I have taken care to use the first person singular, in order not to evade the responsibility. I wish it to be clearly known that the Sub-Commission is in no way responsible for the present report. I and I alone am responsible for everything contained in it.

Before concluding, there still remains for me the pleasant duty of thanking, in the name of our Sub-Commission, the Department of Education for the liberality, with which it has supplied us with the means of executing our work.

R. FUJISAWA.

Tokio, *June* 1912.

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

JAPAN—Delegate: R. Fujisawa (Tokio)

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T[aneyasu] Gotō, *Teacher at the Tokio Higher Normal School.*

T[oshirō] Andō, *Teacher at the Tokio Higher Normal School.*

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Miss Kimiko Horiguchi, *Teacher at the Tokio Higher Normal School for Women.*

T[adaroku] Ōtashiro, *Lecturer at the Tokio Technical College.*

T[ōjurō] Okada, *Teacher at the Aoyama* [*Tokio*] *Normal School.*

LIST OF DIVISIONAL REPORTS

[Bound together in one separate volume]

Article I.——The Teaching of Mathematics in Elementary Schools. Prepared by T. Andō and T. Gotō, *Teachers at the Tokio Higher Normal School,* under the direction and supervision of T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

Article II.——The Teaching of Mathematics in Middle Schools. Prepared by N. Nishikawa, *Professor at the Tokio Higher Normal School,* under the direction and supervision of T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

Article III.——The Teaching of Mathematics in Higher Middle Schools (High Schools). By D. Sudō, *Professor at the First High School,* Dr. K. Shibata, *Professor at the Tokio Imperial University,* and T. Kubota, till recently *Professor at the First High School* and now *Assistant-professor at the Tōhoku Imperial University.*

Article IV.——The Teaching of Mathematics in the Faculty of Science (College of Science) **of Imperial Universities.** By T. Yoshiye, *Professor at the College of Science, Tokio Imperial University,* and Dr. S. Nakagawa, *Assistant-professor at the College of Science, Tokio Imperial University.*

Article V.——The Teaching of Mathematics in the Faculty of Technology (College of Engineering) **of the Tokio Imperial University.** By Dr. K. Shibata and Dr. S. Yokota, *Professors at the College of Engineering, Tokio Imperial University.*

Article VI.—The Teaching of Mathematics in Normal Schools (for preparing Male Teachers for Elementary Schools). Prepared by T. Okada, *Teacher at the Aoyama (Tokio) Normal School*, under the direction and supervision of T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

Article VII.—Training of (male) **Teachers for Intermediate Schools.** By T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

Article VIII.——The Teaching of Mathematics in Girls' High Schools. Prepared by Misses Yoshi Ogawa and Kimiko Horiguchi, *Teachers at the Tokio Higher Normal School for Women*, under the direction and supervision of I. Mori, *Professor at the Tokio Higher Normal School for Women.*

Article IX.——The Teaching of Mathematics in Normal Schools for Women (for preparing Female Teachers for Elementary Schools). Prepared by Misses Yoshi Ogawa and Kimiko Horiguchi, *Teachers at the Tokio Higher Normal School for Women*, under the direction and supervision of I. Mori, *Professor at the Tokio Higher Normal School for Women.*

Article X.—The Teaching of Mathematics in Higher Normal Schools for Women. By I. Mori, *Professor at the Tokio Higher Normal School for Women.*

Article XI.——The Teaching of Mathematics in Commercial Schools and Colleges. By G. Sawada *Professor at the Tokio Commercial College.*

Article XII.——The Teaching of Mathematics in Technical Schools and Colleges. Prepared by T. Ōtashiro, *Lecturer at the Tokio Technical College*, under the supervision of M. Mimori, *Professor at the Tokio Technical College*, and S. Yokota, *Professor at the Faculty of Technology, Tokio Imperial University.*

Article XIII.——The Teaching of Mathematics in Schools under the Army Department. By Dr. A. Imamura and S. Fujita, *Professors of the Army Department.*

Article XIV.——The Teaching of Mathematics in Schools under the Navy Department. By S. Ōba and S. Shimizu, *Professors of the Navy Department.*

Article XV.——The Teaching of Mathematics in Schools under the Department of Communications. By K. Asakoshi, *Professor at the Mercantile Marine School.*

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SUMMARY REPORT

on

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in

JAPAN

PREAMBLE

The present paper is intended to give a *résumé* of the reports to be prepared by the various divisions of the Japanese Sub-Committee of the International Commission on the Teaching of Mathematics, which will be briefly designated as the *Divisional Reports*. It has, however, been made a little more detailed than a mere summary, in order to cope with the uncertainty as to the forthcoming of some of the divisional reports, in view of the lack of time, likely to be most keenly felt in connection with the work of translating the Japanese original into English. Besides, it contains some matters which are presumably not found in the divisional reports. We premise some general features of our educational system, and a schematic survey of our educational institutions, their sequence and interrelations.

CHAPTER I

GENERAL EDUCATIONAL SYSTEM

One of the prominent features of our educational system, if not the most prominent, is the almost absolute centralisation of authority in educational matters. It obtains perhaps even to a greater extent than in Prussia, and contrasts with the state of educational affairs in America, where, as it seems, much liberty is left to voluntary initiative in the absence of authoritative central legislation and supervision. The unification along certain lines, which, together with its consequent inflexibility, may often be ultimately traced to this cause, has naturally its advantages as well as disadvantages. It seems as if its advantages alone have hitherto been pointed out; but, now, the day is dawning when its disadvantages will be discerned side by side with its advantages.

The administration of educational affairs is under the Department of Education. The head of the Department is the Minister of Education, who is a member of the Imperial Cabinet and is directly or indirectly in charge of every thing pertaining to education. He is assisted by a vice-minister, several directors of bureaux, and a number of councillors and secretaries.

It may not be superfluous to give a glimpse of the history of education in Regenerated Japan, which reaches not much further back than some forty years. The Department of Education was first definitively established in September 1871, after having passed through a protoplasmic evolution lasting a couple of years. In the following year was issued the first Educational Code consisting of one hundred and

nine articles. It was accompanied by a sort of preamble, the gist of which is that all people, high or low, and of both sexes, should receive a general education, so that there should be found none in the whole Empire, ignorant and illiterate. And thus was laid the foundation of compulsory education. The Code stipulates that the whole country should be divided into 8 *Large-educational* districts, each with a University. Each of these districts was to be subdivided into 32 *Intermediate-educational* districts, each with a Secondary School, every one of which was in turn to be further subdivided into 210 *Small-educational* districts, each with an Elementary School.

The bold scheme of educational organisation embodied in this Code, which is often said to have been modelled on the French system, could not possibly be carried out in its entirety. Insuperable difficulties were found in the lack of men and means to execute it. Nevertheless, as concerns elementary education, its essence was being gradually put into actual practice and real existence. And in contrast to the higher education of our own days on which it has left but slight traces, it is only right to consider the Code as the root, from which the present organisation of our elementary education has sprung. Elementary schools were started without delay; and, in less than a year after the promulgation of the Code, more than one thousand such schools were already in existence. Ever since, their number has steadily increased year after year. At present, there are probably about twenty-seven thousand elementary schools, with an aggregate of nearly one hundred and fifty thousand teachers and seven million pupils. How far we have been successful in this direction, may be judged by the percentage of the number of children actually attending school in the total number of children of school age, which runs as high as 98%.

The interval which separates the present day from the

date of the promulgation of the Code, witnessed many changes and various reforms, and, among them, particularly, those relating to secondary education, were of a sweepingly revolutionary character, and, in some instances, were afterwards found to be altogether impracticable. For the details thereof, which are very instructive, we refer to the works cited at the close of this chapter. Suffice it to say, however, that on the whole there has been constant progress in one way or another.

We now pass on to the educational system in vogue at present. Quite recently, ordinances were issued, prescribing important changes, in particular, as regards higher secondary education; but as they do not come into force till some time later, and as moreover there is even talk of their indefinite postponement, we shall confine ourselves to the actual status in 1911.

The educational organisation of a country may be compared to a big tree, with the main trunk of a more or less conical shape imperceptibly sloping upwards, and branching off at various heights and in manifold directions. At the base of the trunk lies the **Elementary School.** The course is divided into Ordinary and Higher. The ordinary elementary course which extends over six years, begins at six years of age, and is *compulsory* for all children. Here as elsewhere, we consider normal cases only, as regards the age of children. The higher elementary course of two years which may be continued one year longer, is not a part of the trunk, but is a branch shooting off at the height of thirteen years of age. It is intended for those children, whose parents are desirous of their receiving a little further education without going on to higher studies. As in the sequel we are chiefly concerned with the ordinary elementary education alone, hereafter, for brevity's sake, we will use the word *elementary* in the sense of *ordinary elementary*.

In elementary education, no distinction is made as to

sex. Generally speaking, boys and girls are taught in the same schools and often in the same classes. But here co-education stops; further on, the education of boys and girls becomes distinct. Now, although the higher education of girls, is surely a vital part of the national system of education, it will be at least convenient to consider it as a branch, indeed a big branch, and not as a part of the trunk of the *educational tree*. So, in speaking hereafter of the continuation of the trunk system, we will state, once for all, that we are concerned with the education of boys only.

Immediately above the elementary course of six years, stands the **Middle School.** A middle school has a course of five years, beginning at thirteen years of age, and, as an accessory, it may have a supplementary course of not more than one year. Very often the middle school education is said to be the highest general education. Indeed, this is true in the sense that the course is undivided and no discrimination is made of pupils as regards their future professions. And, although more or less differentiation takes place in schools which follow upon the middle school, still it seems to me that, in substance at least, if not in appearance, our general education stretches a little beyond the limit of middle school education. To this point we shall return later. The qualification of having gone through a middle school is nowadays required not only for entering most of the higher institutions and professional schools, but also in undergoing various examinations leading to vocations of different kinds. The completion of the middle school course is sometimes even required for having access to certain social and other privileges, such as those pertaining to the one year military service. Thus it may not be inappropriate to consider middle school education as socially obligatory, though not legally compulsory, for all those who aspire to membership in that part of the community usually known as the educated class.

Next in order to the middle school comes the **Higher Middle School** whose course extends over three years. It contains several preparatory courses leading to different faculties * of the university, and is thus intermediary between the middle school on the one hand and the university on the other. This school, or, rather the one which corresponds to this on the progressive scale of the educational system, has passed through many changes, some of which were of momentous concern and even touched the fundamentals of its very existence. Its name also was changed several times; sometimes the word *middle* was struck out, and, then, later on, it was again reinstated. Indeed, its present name, literally translated, is *higher school*; but as *middle* is to be inserted betwixt in the near future, and as, moreover, it seems to me to be more befitting to do so, I have named it as above. The history of this kind of school is marked by futile attempts to make it the seat of a higher general education in continuation of middle school education, instead of letting it to remain, as it is at present, a preparatory school for the university. Indeed, the reform in prospect is also of this nature. It seems as if the purpose of this kinds of schools is destined to repeat cycles of fluctuation between the highest general education and the preparatory education for the university. To come to the point, the question at issue converges to this: whether the intermediary school between the middle school and the

* The official translation is *college* and not *faculty*. Here as elsewhere, as regards the translation of the names of schools and the like, I have allowed myself to deviate somewhat from the authoritative ones, as dogged sticking to the official rendering seems to be liable to confuse the reader of this condensed account, giving the sequence and interrelations of so many different kinds of educational institutions. I use the names *faculty*, *college*, *school* (in a specific sense) in the descending order of grade or standard and of the corresponding qualification for entrance; for example, in this sense, a faculty of technology (of the university) is higher than a technical college, which, in turn, is higher than a technical school.

university is to be affiliated to the former or to the latter. On the whole, the tendency seems to lie in the direction of giving more and more preponderance to the aspect of higher general education, in conformity with the requirements of the time which demand a raising of the standard of general education. Even now, when the aim of the higher middle school is avowedly preparation for the different faculties of the university, and the courses are organised with this object in view, looking a little into the details of its curriculum, it is not difficult to detect traces and elements of a higher general education, common to all the courses. On this account, I deem it only proper to consider, that, as was said before, our highest general education stretches, beyond the limits of the middle school course, at least some distance into the higher middle school course. The middle school course and the higher middle school course excluding possibly the last one year taken conjointly, may be looked upon as forming our secondary education, properly to be so called. Hereby we give to the word *secondary* its average significance, as it is used in several of the most important countries of the world.

Next to the higher middle school, and at the apex of the trunk system, stand the different **Faculties of the University**. There are four universities at present, namely, the Tokio, the Kioto, the Kiushiu, and the Tōhoku Imperial University. The last two have been but recently started, and both have only two more or less complete faculties. The Tokio Imperial University is the oldest institution of the kind, and, before arriving at its present condition, has seen several radical changes and thoroughgoing reforms. As it is complete, and its organization stable, at least for years to come, and as, moreover, other universities are on the whole modelled on it, we take it as the typical representative of our universities. It contains six faculties, namely, of Law, Medicine, Technology,

Literature, Science, and Agriculture. These names will hardly convey any very definite idea of their contents. The name "Law" covers law, political science (*Staatswissenschaft*), administrative science (*Verwaltungslehre,*) economics, and the like, and even the so-called highest commercial education; "Medicine" includes pharmacy as an accessory; "Technology," whose official rendering is *engineering,* may most appropriately be replaced by *polytechnics* in the widest sense of the word; "Literature" embraces philosophy, history, literature, and allied subjects; "Science" includes mathematics and science (in a narrower sense); lastly "Agriculture" comprises agriculture, agricultural chemistry, sericulture, forestry, veterinary medicine, and fisheries.

As far as the division into different faculties is concerned, our university system is very much like that of Germany. So, a little comparison may not be out of place. The faculty of law corresponds to a *rechts-und staatswissenschaftliche Fakultät*; the faculty of medicine to a *medicinische Fakultät* with the course in pharmacy annexed; the faculty of literature to a *philosophische Fakultät* in the narrower sense, or *philosophisch-philologisch-historische Abtheilung* of the wide-embracing faculty bearing the same name; the faculty of science to a *mathematisch-naturwissenschaftliche Fakultät* with the course in pharmacy cancelled; the faculty of technology to a *technische Hochschule* or *Polytechnikum* of the highest standing, together with a *Bergakademie,* merged into a university and forming one of its faculties; lastly, the faculty of agriculture* to a *Hochschule für Bodencultur* with a wider range of contents than the one in Vienna, or the aggregate of a *landwirtschaftliche Hochschule,* a *Forstakademie,* a *tierärztliche*

* Some of the subjects taught at this faculty are to be found in the philosophical faculties of a few German universities, such as Halle a. S. and Leipzig.

Hochschule and the like raised to the standard of, and incorporated as a faculty into, a university. The wide-embracing character of our university just elucidated, would justify our having considered it as forming the topmost part of the main trunk of our *educational tree*.

We have just noticed the similarity existing between a German university and ours, as regards division into faculties. Here, however, the similarity ceases, as a matter of course, not in the subject matter of instruction, but in the spirit which governs the method of teaching. Generally speaking, each faculty is divided into a number of sections, each section having a distinct course. Thus, for example, the faculty of science contains seven principal courses, namely, of Mathematics, Astronomy, Physics, Chemistry, Botany, Zoology, and Geology. Now, every course has a prescribed curriculum and no choice is allowed, except as regards a few so-called optional subjects. For the details of the different courses in each faculty, we refer to the Calendar of the Tokio Imperial University. Only we may add, that, in the faculties of law and of medicine (excepting pharmacy), the course extends over four years, while, in the remaining faculties (and also in pharmacy), it covers three years.

A student entering a faculty of the university choses his course, and then he must adhere to it. Sometimes a course breaks up into two after some time. For example, the course of biology splits into the two courses of zoology and of botany. In such a case, of course, the student has to make his choice again. In every case, however, he has to pursue his studies according to a fixed programme, attending predetermined lectures, going through laboratory or seminary work, or practice as prescribed, and finally graduating at the close of three (or four) years, provided he has successfully passed the annual examinations and the graduating examination which is often accompanied by a

thesis or essay. Thus he is taught much in the same way as in a middle school or in a higher middle school, save that the subjects taught are specialized and very far advanced. All this appears in striking contrast to the *akademische Freiheit*, that is, an almost absolute exemption from control of any kind, enjoyed by a German university student, which includes, among others, the freedom of attending whatever lectures he pleases. Our university student is, so to speak, less conscious of his own importance than his German comrade. He is forced to learn by prescribed regulations and by surrounding circumstances, while his German friend waits till he wishes to learn and then studies hard to the best of his ability, his motive in so doing coming from his own free will. Of course, we are speaking in a broad sense, in thus indicating the characteristic difference in the general features of the two kinds of university education. Both ways seem to possess advantages as well as disadvantages. Our way is effective in bringing the aggregate total of students to a certain state of qualification and proficiency, while the other way has the disadvantage of leaving mediocre students to remain where they are, and the advantage of inducing more gifted ones to go on ever higher into the domain of original research. What has been just said is only true of the present condition of our university education. There is a strong tendency to combine the two systems; indeed, some attempts are being made in this direction.

The aforesaid character of our university education is mainly due to its being a sort of professional education, by which is to be meant the education leading directly to some kind of profession. In fact, the university graduates enjoy many privileges leading to various professions: thus those of the faculty of law have special privileges in connection with examinations for becoming barristers, or judges, examinations for higher civil service, etc.; those of the faculty

of medicine are entitled to a licence for medical practice; those of the faculties of technology and of agriculture can at once become officially recognized experts in their respective lines of speciality; those of the faculties of literature and of science can get a licence for becoming teachers of special subjects in a middle school; and so on.

A graduate of the university is entitled to call himself *gakushi* (bachelor?). This name is often thought to be a sort of academic degree, but, in reality, it is not so. It is simply a condensed epithet equivalent to "graduate of a university faculty." Now, to become a *gakushi*, is not only the ambition of the majority of our boys, but it is the goal towards which parents are eager to see their children striving. This is the natural outcome of the so-called family system of our social structure. Very often parents are willing to sacrifice the very last of their possessions for the education of their children, expecting, in turn, that the children, on becoming *gakushi*, would be able to support not only themselves but their entire family.

I have dwelt somewhat at length on our university education, perhaps longer than would seem necessary in this report which has to deal chiefly with the teaching of mathematics in secondary schools. But such is not the case. To the characteristic peculiarity, just spoken of, of our social structure, and to the professional character of our university education, are to be attributed some of the specific features of our secondary education. Even the teaching of mathematics in secondary schools is influenced more or less by the same cause.

Any one who has passed through a higher middle school has the right to enter a university. The university must promote everybody, even the least capable, who have been matriculated, and send them forth to society at large as experts in some speciality, unless it goes to the extremity of dismissing them altogether as incapable. Under such conditions,

it is only natural that the university should exert paramount—even dictatory—influence on secondary education. It seems as if, in other countries, the secondary education comes first, so to speak, and the university education accommodates itself to it. With us, it is otherwise; it is the university education which reigns supreme and the secondary education is subordinated to it. As a result of this, there is a strong tendency to push into, and heap up in, the curriculum of the higher middle school course, the materials of instruction, which bear even the slightest character of being preparatory for the dominating professional courses in the university, and which, in other countries, are usually taught at the university itself. As an example, all the essential parts of differential and integral calculus, together with the analytical geometry of space, are taught in a higher middle school. Before leaving this point, however, we have to notice the presence, among us, of educationalists who are strongly in favour of freeing our secondary education from the undue influence exerted by the university. The contradictory influences thus brought to bear on secondary education, some traces of which might perchance be found in other countries too, are here felt most keenly, and account for some inconsistencies observable in our secondary education.

Let us now return to the higher middle school. There are three preparatory courses for the university; the first is for those who wish to enter the faculties of law, or literature; the second for those who wish to enter the faculties of technology, science, or agriculture, or the section of pharmacy in the faculty of medicine; the third for those who wish to enter the faculty of medicine proper. According to the reform in prospect, which is to take effect in a short time, there are to be only two separate courses, which, compared ~~to~~ with the present courses, lose not a little of their character of being preparatory for the university.

The trunk of our educational system thus consists of the elementary school, the middle school, the higher middle school, and the university. Below, there is the Kindergarten, and, above, the University Hall for post-graduate studies and original investigations; but they may be left out of consideration in speaking of the trunk of our *educational tree*. Also for the present, we exclude the university and the last one year of the higher middle school. Now, six years of the elementary school beginning at six years of age, five years of the middle school, and the first two years of the higher middle school, in all about 13 years,* would roughly correspond, on the average American scale of educational gradation, to eight years of the primary school, four years of the high school, and one year (freshman year) of the college, in all 13 years; and, on the German scale, to the first three years of the *Volksschule* (or an equivalent preparatory course for the *Gymnasium*), and nine years of the *Gymnasium*, *Realgymnasium*, or *Oberrealschule*, in all 12 years. In making such a comparison, one must not overlook the two greatest difficulties of all connected with our education:—viz. the extreme complexity of our own language which is in reality two languages blended together, and the great difficulty of learning two foreign languages, English and German or French, which, as it seems, far surpasses the difficulty experienced by people using the Roman alphabet in learning Latin and Greek.

In the higher middle school, and the university, the academic or school year begins on the 11th of September and ends on the 10th of July, with a summer vacation of two months intervening. In the middle school, and the elementary school, and also in most of the other kinds of schools not included in the trunk system, the school year begins on the 1st of April and ends on the 31st of March;

* In reality, a little over 13 years, as will be explained immediately.

it is divided into three terms, the first extending from April to the summer vacation, the second from September to the end of the year, and the third from January to the end of March.

The difference in the school year of a middle school, and of a higher middle school, gives rise to an intermediate space of about four months. This is the hardest time for one who wishes to enter the upper school. He has to make preparations for the competitive entrance examination, which is often spoken of as deciding his destiny. In the prospective reform plan already referred to, this gap is to be filled up by beginning the higher middle school in spring and cancelling its last year, so that it will, then, have a course of about two years and four months.

Besides national holidays, of which there are about ten, but some of which lie within vacations, a school has usually a holiday commemorating some event of its own, as its foundation and the like, and, sometimes, a few more of local or casual significance. In the middle school, summer vacation lasts a little more than a month, and in the elementary school about one month or even less. In both kinds of schools, winter vacation lasts about two weeks at the end of the year and the beginning of the next; spring vacation also about two weeks at the end of the school year and the beginning of the next. Again, some days are usually allotted for *instruction excursions*, during which pupils are expected to learn many things from observation by visiting different places under the guidance of their teachers, or, to *marching excursions*, during which military gymnastics and drill are taught and exercised on a greater scale than is possible on the play-ground attached to a school. Thus the number of working days is, in most cases, a little over 200 days.

The number of hours per week increases gradually with the years in an elementary school, reaching a maximum of

about 28, and is 30 or thereabout in a middle school. An hour usually means 50 minutes, there being a pause or recess of about 10 minutes either at the beginning or at the end of it. In the lower grades of an elementary school the working part of an hour is even less than 50 minutes; and sometimes hours are split into halves with intertwining pauses.

As was said before, we are going to consider only the normal case, as regards the age of boys. Let us add here, once for all, a few words concerning the actual situation. Owing to the insufficiency of schools, boys are often required to pass through competitive examination in going from an elementary school to a middle school, in addition to the one for entering a higher middle school. Moreover, as is only natural, some of them may not be able to finish a school course in the prescribed number of years. Thus it happens, that a student reaches the age of thirty or thereabout, by the time he has finished his university course. Such cases are unfortunately not rare. And this evil is, by no means, restricted to the trunk; but similar things happen in some of the branches of our *educational tree*. This is looked upon in some quarters as an alarming feature of our education, and remedies are being sought to alleviate this evil. It seems to me that the root of the evil lies in the professional character of our university education and the consequent undue influence exerted by the university on our secondary education.

Having done with the trunk, it remains for us to look at some of the more important branches of our *educational tree*, such as girls' schools, normal schools, technical schools, and so on. They will be dealt with, each in a subsequent chapter treating of the teaching of mathematics in the respective kind of schools. On every such occasion we shall refer to the trunk system considered in this chapter, and explain, where and how it branches off the main trunk.

It is hoped that the sequence and interrelations of various kinds of schools are, thus, rendered clearer.

Before closing this chapter, however, I may be permitted to refer once more to the almost excessive centralization of authority spoken of at the outset. It seems to exert an important influence, though in an indirect way, upon the progress of the teaching of mathematics, perhaps even more important than would appear at first sight. That mere change is not identical with bettering or real improvement, is too obvious to be mentioned; and yet, looking back into the history of the past reform movements, one can not but be struck with the fact of how often so obvious a truth has been lost sight of. Many of the reforms which have subsequently proved to be real improvements, originated in individual initiative. Somehow the idea of a reform scheme suggests itself to somebody. He thinks it over a long time, communicates its gist to his fellow thinkers, and sometimes, when possible, tests its soundness by appealing to experiments within his environment. Sometimes he has to fight against adverse views. Needless to say that it requires devotion and perseverance in all this. As he becomes more and more convinced of the soundness and feasibility of his scheme, he becomes its zealous advocate and the nucleus of a reform movement. Often such nuclei having the same or nearly the same object, are found here and there, not only within one country, but, in some instances, scattered over different countries, at first isolated, but gradually becoming cognizant of their co-existence. In most cases, the schemes proposed may not be exactly alike, but they have a common basis and similarity in details. They are then discussed and resolutions passed in gatherings of men competent to judge such affairs. It is thus that a scheme ripens. Not only is it desirable but even necessary, that any reform scheme should go through some such stages of development and consolidation prior to its final adoption.

Now the excessive centralization of authority, which, now and then, indulges in drastic measures and sweeping legislation, is liable to the risk of adopting, or sometimes even dictating, reform schemes which, in due course of time, may turn out to have been hasty and immature, or, altogether impracticable.

New Japan was born some odd fifty years ago. Since the Restoration of 1868, She has been very busy in establishing order in the chaos, created by pulling down whatever was old, under the influence of contact with the outside World. In education, the last forty years may be characterised as the period of transition, during which we have witnessed so many revolutionary changes and radical reforms, unparalleled in the history of education of any other country. Too rapid a succession of changes or reforms, without awaiting the ripening of the fruit of the preceding one, although some of them might have been inevitable under the circumstances, and, each taken by itself might be a wholesome measure, can not fail to prove, in the long run, to be detrimental to the real progress in such matters as education, and, not seldom, leads to that state of confusion which bristles with difficulties and contradictions. At present we have either passed through, or are nearing the end of, this period of transition. It is now time to look forward to some sort of stability. To this incidental circumstance, is due, that traces of some conservatism are likely to be found, here and there, in this report, especially in those parts which deal with future tendencies.

The chief works which may be consulted in connection with the contents of this chapter are the following:—

An Outline History of Japanese Education, Literature, and Arts. Prepared for the Philadelphia International Exhibition, 1876, and reprinted for the Paris Exposition, 1878.

Notice sur L'Organisation actuelle de L'Instruction publique

au Japon. Préparé pour L'Exposition universelle de Paris, 1900.

Education in Japan. Prepared for the Louisiana Purchase Exposition, 1904.

Education in Japan. Prepared for the Japan-British Exhibition, 1910.

The above four publications, issued by the Department of Education, are not on sale and mostly out of point.

Japanese Education. Lectures delivered in the University of London. By Baron D. Kikuchi. Published by John Murray, Albemarle Street, London, W., 1909.

Of this work, it will be difficult to speak too highly. It is the first systematic and intelligible exposition of education in Japan, ever offered to the World at large, and is unsurpassed by any thing which has hitherto appeared in print. Long will it remain the standard work on the subject which its treats. An article under the caption "Education in Japan" by the same author in the Japan number of the London Times (July, 19, 1910), and also an article entitled "National Education in the Méiji Era" by Marquis Saionji, the present Prime Minister, and, sometime Minister of Education, forming Chapter XIX of the second volume of "Fifty years of New Japan" (published by Smith, Elder, & Co. 15, Waterloo Place, London S. W., 1910) may also be consulted to advantage.

CHAPTER II

THE TEACHING OF MATHEMATICS IN ELEMENTARY SCHOOLS

SOROBAN-CALCULATION

The only branch of elementary mathematics taught in the elementary school is **Arithmetic.** Now in different countries the word *arithmetic* has a significance somewhat varied as to its scope. With us it means nearly the same thing as what is usually known in England and America by the name of practical arithmetic. There is a certain kind of so-called useful knowledge closely connected with the art of calculation, which is necessary in daily life and consequently deemed worthy to be included within the limits of general education. Such is e.g. some knowledge relating to weights and measures, measurements of time, coinage, interest, bonds and shares, and the like. All this is included in our arithmetic, not as an accessory, but as forming an essential part. Moreover, in this respect, our arithmetic is perhaps a little more comprehensive than that of the English or American type. Thus, although our arithmetic is conventionally a part of elementary mathematics, in reality, it is something more. Indeed, that it is something more than the mere art of computation is one of the characteristic features of our arithmetic.

Not the least noteworthy feature of our arithmetic is the concomitant use of two kinds or sorts of calculation. One of these is the ordinary written calculation. For the other,

as there is no translation hallowed by usage, I propose the name "soroban-calculation" or "beads-calculation." This latter is the calculation performed by means of a *soroban*, a primitive instrument, which is often called abacus, but for which I prefer to retain its original name. As to the former, the name *written calculation* is apt to suggest its contradistinction to mental calculation; so, wherever it is thought desirable to lay stress on its distinguishing characteristics as opposed to beads-calculation, we shall call it "brush-calculation," where the word *brush* is used in the wider sense of anything whatever wherewith we write.

It is now our purpose to give some account of beads-calculation. We premise a few words relating to the so-called mathematics of the old Japanese school.

That there is such a thing as the mathematics of the old Japanese school, is now pretty well known. On the occasion of the second International Congress of Mathematicians, held at Paris in August 1900, I had the honor of laying before the section of Bibliography and History a short paper entitled "Note on the Mathematics of the Old Japanese School" which is to be found on pp. 379—393 of the *Compte rendu du deuxième congrès international des mathematiciens*, published by E. Duporcq, Paris, 1902. In this connection, our thanks are due to Professor Paul Harzer of the Kiel university for his erudite address entitled *die exakten Wissenschaften im alten Japan*, delivered in 1905, on the occasion of the German Emperor's birthday. It contains a critical examination of some of the more important parts of this peculiar sort of mathematics, based upon the facts given in the *Anmerkungen* appended to the printed form of the address.* We can not help admiring the care and painstaking accuracy, especially on the part of one who is not conversant with the language of the original, in collecting

* Published in the form of a monograph by Lipsius & Tischer, Kommissions-Verlag für die Universität, Kiel, 1905.

the materials therein given from scanty sources, and in digesting them so well as he has done.

A succinct exposition of this kind of mathematics is to be found in an interesting paper by T. Hayashi, written in English under the captain "A brief history of the Japanese mathematics," which appeared in the Dutch journal, *Archief voor Wiskunde* (2nd series, Vol. VI, pp. 296—361, 1905, and Vol. VII, pp. 105—163, 1907). It is, as far as I am aware, the first attempt to give, in a language other than that of the original, something like a systematic account of this peculiar form of mathematics. A short note by Y. Mikami, giving some corrections and addenda to the above, has since appeared in the same journal.

Thanks to the labours of Baron D. Kikuchi, Prof. T. Hayashi, Mr. Y. Mikami, and others, the mathematics of the old Japanese school, have been, though in a fragmentary form, made accessible to mathematicians of other countries. Their papers are to be found scattered through the volumes of the Proceedings of the Tokio Mathematico-Physical Society, of which there are two series, *Archiv der Mathematik und Physik*, *Bibliotheca Mathematica*, and the Dutch journal just referred to. Some account of it is also to be found in the well-known monumental work of M. Cantor, *Vorlesungen über die Geschichte der Mathematik*.

We are not going to give even a superficial account of this mathematics. The bare fact that there is such a thing as the mathematics of the old Japanese school, is sufficient for the purpose of this report. In days of old, they were studied and cultivated by a select few for no other purpose than for their own sake. To the larger public, their zeal and devotion might have appeared in the light of curiosity or even whim. Indeed, of this mathematics, no trace is discernible in the education, properly to be so called, of the days prior to our Renaissance. At that time, education which was often mixed up with learning, was considered to be a monopoly of

the upper class of people who were proud of being called *samurai*—the name which means membership of the two-sworded caste and inspired the consciousness of superiority over the mass of the people including farmers, artisans, and tradesmen. The old education was split into two, the physical and the intellectual. The physical education consisted in instructions in fencing, archery, handling of spears, horse-back riding, and the like, with which was associated a part of moral training through the medium of Spartan discipline and knightly bearing insisted upon in practicing these arts. The intellectual education was essentially classical in nature, and profoundly influenced by the Confucianism tinged with the chivalrous spirit of feudal times. To do such things as calculation was thought condescending beneath the dignity of the *samurai* rank. They even went to the length of boasting of their ignorance in the art of computation practiced by trades-folk, who were looked upon as an inferior class of people.

Let us now return to the beads-calculation, which is the first rudiment—in fact the only arithmetic—of the mathematics of the old Japanese school. Merchants and tradesmen had to make themselves masters of it from the sheer necessity of their profession. They learned it in the same spirit as a farmer learns ploughing or a carpenter dovetailing. Special stress was laid on the quickness of operation which depends mainly on the alertness of the fingers-tips' movement on a *soroban*. Among a mercantile people, quick calculation became such a favorite practice that it was not only their indispensable business but served for their pastimes. Calculation tournaments were very frequent. Competitors were gathered, each provided with a *soroban;* a referee gave the signal to start and then, as fast as he could, read off numbers to be added; instantly the whole room resounded with the rattling of beads; those who were not able to follow dropped off one by one, and the clicking noise became

feebler little by little; at the end of half an hour or thereabout the reading out of numbers was stopped abruptly; right away the sums obtained were inspected; and those who got the correct result came off as victors. Usually it was addition; but other kinds of calculation were also practiced and proficiency tested in the same way. Besides, among experts who were conscious of their own adroitness and longed to boast of it, even a sort of challenge was not uncommon. In this way quick calculation on the *soroban* attained that state of perfection which it would be hard to realize in written calculation. The dazzling dexterity shown by the reflex movement of the finger-tips of a *sorobanist*, which may perhaps be compared to that of the best pianist, is indeed a marvel and defies the imagination of one who has not seen it in actual practice.

The account, thus far given, of beads-calculation relates to a period previous to the Restoration of 1868, during which it was in full bloom though restricted to the narrow circle of mercantile community. Meanwhile it had taken firm root, and manifested its latent potentialities by surviving the stormy days immediately preceding the Restoration and the chaotic childhood of Rejuvenated Japan.

Soon after the Restoration, schools were started. And we found ourselves confronted with the problem: What and how should we teach in these schools? As regards elementary mathematics, we had to choose between that forming a part of what is now known as the obsolete mathematics of the old Japanese school and that of the Western type. As to our choice, I wish to quote my own words forming the conclusion of my paper presented to the Paris Congress of 1900: "It was surely a wise policy on the part of the educational authorities that they, in organising the new system of education, put this mathematics of the old Japanese school entirely out of sight, and were anxious to introduce free and unmolested mathematics which has no schools and whose

universal language is intelligible to all the civilized nations."

Judged by later experience, there is no room left for doubting the wisdom of the choice; but this decision was owing rather to the mania for every thing Western, prevalent at the time, than to anything else. There was a epoch, during which men had their eyes opened only for things occidental, and whatever was old was put aside to make room for something foreign. As Baron Tsuji, perhaps the greatest living authority on our elementary education, told us but recently in his able address on the occasion of the commemoration of the fourty years' existence of the Tokio Higher Normal School, the mere fact that a thing is of Western orgin, was, at that time, considered to be sufficient reason for considering it good and therefore deserving to be adopted.

Education which in olden days was restricted to the *samurai* class, was now extended to all classes of people, including of course merchants and tradesmen to whom beads-calculation was indispensible. Again, non-professional use of the *soroban* had somehow permeated our whole social structure, despite the aforesaid prejudice of bigoted *samurai* against it. All this, however, did not helped to save beads-calculation from sharing the fate of other things handed down from our fathers and forefathers. And the most effective means of extirpating it was found in its rejection from the newly organized elementary education and the exclusive adoption of written calculation. This trend reached its climax when the Department of Education, in accord with the saying, "let the physician be the first to follow his prescription," positively forbade the use of the *soroban* in its own office work. The prohibition had, however, to fight against a real need, and, in due course of time, relaxed into tacit toleration. In elementary education too, beads-calculation was as often reinstated as it was set aside. Meanwhile

it lost its character of being a part of the antiquated mathematics, and metamorphosed itself into an essential constituent of the new arithmetic.

That beads-calculation has successfully resisted the oppressive measures to eradicate it, is no doubt, in the first instance, due to the inertia of long usage; but this does not seem to account for everything. To me it appears that its own intrinsic merit is the main factor in its preservation in some form or other. I myself was, at one time, of the opinion that, for the sake of our education, it should be banished from our schools; but, since then, I had to change my views. That beads-calulation outlived the severe ordeal touching its very existence, seems to bespeak for it a brighter future.

Our *soroban* is, in appearance, very much like a Roman abacus, a Russian *tschotü*, or a Chinese *swan-pan*. Unusually they are all categorically included under the name *abacus*. Now in works on the history of mathematics, we very often meet with statements, such as "it is still in common use in such and such countries," couched in words which seem to imply that it is doomed to become obsolete sooner or later. This sentiment is so deep-rooted as to require courage even to doubt its validity; and yet I wonder if it is not an unwarranted prejudice.

With the progress of civilization, we have to calculate ever so much more. Things formerly thought capable of qualitative discrimination only, have since been made subject to quantitative measurement. Indeed, one of the cardinal features of modern civilization is the tendency to reduce as many things as is practicable to numerical evaluation as much as possible. Now there is a saying about the three R's "reading, writing, and 'rithmetic," which has its popular equivalent in Japanese. It simply signifies, that reading, writing, and arithmetic are requisites of every-day life. Looking back upon the last thirty years, we can not

but be struck by the wonderful progress made in facilities for reading and writing. As to reading, we can nowadays get just that we wish to read; there is more reading matter than we can actually read, and for all that, we are incessantly urged by never-tiring energetic publishers to read more. As regards writing, we need only recall the rapidity with which the use of the type-writer has spread all over the world. In counting or calculation alone, it seems as if we are lagging. Needless to say that here quickness inseparably coupled with correctness and involving the least amount of mental work is the desideratum. This would naturally suggest looking into calculating machines. As early as the days of Pascal and Leibnitz there was felt the need of such an instrument, and calculating machines of different kinds were actually constructed; but even the most celebrated machine of Babbage has passed into historical oblivion. Modern calculating machines are models of human ingenuity; yet this very ingenuity bids fair to be a drawback for their ever acquiring an importance in counting at all comparable to that of type-writers in writing. Calculating machines, thus far invented, appear to have but little chance of being applied in the wider field of every-day requirements. If so, the question arises: How are we to find that something which performs for calculation a service similar to that of the type-writer in writing? It is a vital question which touches the weal of the whole of humanity and has an important bearing on general education, in particular, on the teaching of arithmetic in the elementary education of any country.

Now after all, is it not possible that our *soroban* happens to be just the thing needed? Who knows whether it is not another example of things long searched for and found at last in the corner least surmised! First of all, it is imperatively necessary to discriminate the *soroban* from the cluster of somewhat similar instruments, usually included under the

generic name of abacus. It was in view of this circumstance that I have persistently retained its original name, in spite of the fact that it is often carelessly translated abacus. No less necessary is it to dissociate it altogether from the simple device consisting of beads stung on wires, which is used in teaching the rudiments of arithmetic in the lowest grades of elementary schools. Surely it requires an effort, but we must try to get rid of the prepossession of considering it as something very primitive or even childish, and look at it in earnest as having possibly a future of its use spreading thoughout the civilized world.

Experiments were often made in order to test the relative merit, as to quickness, of beads-calculation *versus* written calculation, not seldom under preconceptions unfavourable to the former. Usually addition was tried, it being the commonest sort of calculation and of the most frequent occurrence. When numbers are given written in ideographs, the advantage of beads-calculation is at once obvious. Only with the introduction of the occidental way of keeping books with Arabic figures, the question arose as to the relative superiority of the two kinds of calculation. Experiments as to quickness have in most cases resulted in favour of beads-calculation; but there is always room left for putting in some sort of objections, such as the individual aptitude of the competitors which it is well nigh impossible to compare, the details of the conditions under which the trial was made, and the like. However, so much is certain that beads-calculation requires far less head work, especially as regards remembering even for an instant partial sums, than written calculation. Again, in shops, or offices or bureaux where calculations are most frequently done, there are usually a number of persons working together; one of them calls out the numbers from the book as fast as he can while two or more do the sums on the *soroban*, so that the result can be verified as soon as the sum is obtained, whereas, in written

calculation, we have to go over the process again into order to verify the result.

Taken all in all, the main factor in deciding this question hinges on the facility or difficulty of acquiring the necessary agility of one's finger-tips on the *soroban*. Now there are some people whose fingers are, as it were, all thumbs, just as there are some who are colour-blind; but any ordinary person has the inborn capacity of his finger-tips acquiring, by a little practice, the requisite nimbleness on the *soroban*, especially if the first impetus has been given in childhood. Often we are told, although I can not vouch for it, we Japanese are a people especially clever with our finger-tips. Perchance it may be argued that what has been just said, applies to us only and not to other people. Against this may be mentioned: among foreigners either now, or at one time, residents here, there is no lack of examples of such who, becoming interested in the *soroban*, by a little patience, have mastered beads-calculation as well as the experts of our own nationality.

There is another feature of beads-calculation, which it may be worth while to notice. The use of the *soroban* is as common in our country that it cannot but attract from the very first the attention of a newly arrived foreigner. He may go shopping, and, at a shop which had the luck to catch his fancy, he buys a few trivial articles and figures up in his own mind how much he has to pay in all. But the shop-keeper will not take the trouble of going through a mental calculation however simple. At once he grasps his *soroban*, adjusts it by a tilt and a rattling sweep of the beads, makes a few clicking moves of his finger-tips on it so quick as to puzzle his customer, and names the price. It is only naturnal that the foreigner, conscious of his own power of mental addition, looks at the tradesman with some slight contempt. But a little reflection should tend to transform this contempt into something like admiration. In

fact, the tradesman can calculate mentally just as well as, or perhaps even better than, his customer; only he will not. If he makes use of the *soroban* in the simplest possible kind of addition or subtraction, that is only an illustration of the irresistible force of habit, and we can only blame him for his being strictly consistent. Are we not apt to make slips and to be tripped up in matters too small to rouse our earnest attention? Machines, too simple to get out of order, are they not more trustworthy than the average human brain, crowded nowadays with the perplexities of mordern civilization? The world is becoming more and more busy and supreme efforts should be made to economize our mental work as much as possible. To one who sees the matter in such a light, it would seem odd that the use of the *soroban* has not spread beyond its ancient territory. This may be partly accounted for by the lack of stimulating novelty, there being similar instruments already known to foreigners. Now, we have learned so much from the West; sometimes we hear it said nowadays that there are things which the West might get from us. It may be compliment or persiflage; but I have audacity to take it *ad literam*. I am not bold enough to suggest immediate adoption without further enquiry, but I will go so far as to assert that our *soroban* is at least worthy of consideration as to its possible introduction into the countries to which we owe everlasting gratitude.

In this connection, I wish to quote a few passages from a paper by my friend and former colleague, Dr. C. G. Knott, now of the Edinburgh University. It is entitled "the Abacus, in its Historic and Scientific Aspects," and is to be found on pp. 18—72 of Volume XIV of Transactions of the Asiatic Society of Japan, published in 1886. He says: "It may be safely asserted that even in the simplest of all arithmetical operations the *soroban* possesses distinct advantages over the mental or figuring process. In a competition in simple addition between a *Lightning Calculator*, an accurate and rapid accountant, and

an ordinary Japanese small tradesman, the Japanese with his *soroban* would easily carry off the palm." Further on he adds: "Let us suppose that a purchaser has bought three articles which are priced at Yen 1.25, Yen 2.89, and Yen 3.17 respectively. How many people out of any hundred of ordinary intellect could add these three numbers correctly in their mind? A Japanese shop-boy with a *soroban* in hand will do it as fast as the numbers can be named, and with greater precision and certainty than many of us could attain in figuring. Fact like these suffice to give to the instrument a certain respectability."

His paper is divided into two parts. The first part which treats of the historic aspect of the *soroban*, may be looked upon as supplementing or corroborating a very interesting and learned essay by Sir E. Clive Bayley entitled "On the Genealogy of modern Numerals," which appeared in the Journal of the Royal Asiatic Society of Great Britain and Ireland, new series, Volume XIV (1882), pp. 335—375, and Volume XV (1883) pp. 1—72. In passing, we may observe that Bayley's article was published but a few years later than the first edition of the first volume of M. Cantor's well-known history of mathematics. To the students of the history of the art of calculation, I can not do better than recommend the perusal of this first part and its parent article just referred to. Dr. Knott, after tracing the evolution of beads-calculation to the present state of perfection, closes the historical part with a eulogy culminating in the words, "the beauty of action of the Japanese *soroban*."

CALCULATION ON SOROBAN

The second part of Dr. Knott's paper treats of the actual method of calculation on the *soroban*. So far as I am aware, it is the only article in any foreign language, which gives a tolerably full explanation of beads-calculation. However, as it would be hardly accessible for many of the readers of this report, who might happen to take interest in the *soroban*, I shall

give, in the following, some account of how calculations are actually done on this unpretentious instrument.

The appearance of the *soroban*, in common use nowadays, is depicted below:

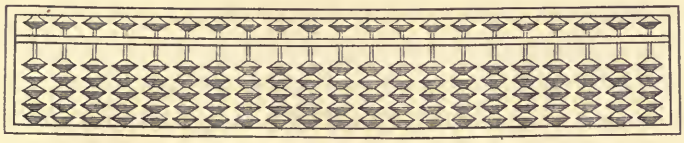

Fig. 1.

A shallow rectangular framework, usually made of hardwood, is divided lengthwise by a narrow strip of wood into two compartments, of which the upper one has a width somewhere between one-third and one-fourth of that of the lower one. The narrow piece of wood, just spoken of, which serves as a partition, will be called the *partition-strip*. It has usually on its upper edge a thin covering made of bone or ivory. Slender bamboo rods which serve as carriers of movable beads, are placed at equal intervals apart through the partition-strip near its upper edge, and firmly fixed into the longitudinal sides of the frame. The beads or counters, usually made of hard wood, have the form of an edged circular disk, i.e. the form of two equal flat cones joined at the base and truncated at their vertices, with a cylindrical hole through the axis. This doubly conical form of the counters is worthy of serious attention; to it more than to anything else, is owing the alertness of operation, the finger-tips easily catching the sharp rim of the beads which run smoothly along their carrying rods. The interval between two neighboring rods is just a trifle greater than the diameter of the circular ridge of a bead. The number of rods varies, and the length, compared to the width, of the framework varies correspondingly. As to the actual size, we have all varieties from the pocket *soroban* to the comparatively big one for school work. On each rod there is one bead in the

upper compartment and five in the lower, the former counting as 5 and one of the latter counting as 1, so that we can count 10 on each rod. For the sake of convenience, we shall provisionally call a bead in the upper compartment, which counts as 5, a *pentaroid*. Let all the *pentaroids* be pushed up as high as possible, so that they touch the upper wall of the frame, and all the beads in the lower compartment pushed down as low as possible, so that the lowest ones touch the lower wall. The same thing might be more compactly expressed by saying:—remove all the beads as far away as possible from the partition-strip. In actual practice all this is done very simply. Tilt the framework so that all the beads run down as low as possible; restore it to its original horizontal position so gently that no bead moves during the process; and then pushing a finger-tip between the partition-strip and the *pentaroid* at the left extremity, let it sweep along the partition-strip to the right end. In the configuration thus obtained, the *soroban* is ready for action, no number being recorded or set on it.

The mode of setting a number on a *soroban* is in principle precisely the same as the Arabic notation, the only difference lying in this, that here we indicate a digit by the position of beads on a rod instead of writing down figures. By a digit we usually mean one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9; sometimes zero may also be included herein, although we shall not go to the trouble of specifying it in each case. One of the rods is chosen for the *units*, the next to the left is the *tens*, next but one to the left is the *hundreds*, and so on. Similarly the successive rods to the right of the *units* rod represent decimal places in the descending order. A digit is set on a rod by moving the corresponding beads as close as possible to the partition-strip.

Thus, for example, to set 3, the three uppermost beads out of the five in the lower compartment are moved upwards until the top one strikes the strip; and to set 7, the *pentaroid* is moved downwards and the two uppermost beads

upwards, so that they, connected by a small portion of the partition-strip, form a whole consisting of three beads.

The amplitude of the movement of a bead is obviously the same for all the beads in the lower compartment, and is equal to the length of the portion of a rod left bare below the partition-strip when all the five beads have been pushed downwards. This amplitude is of primary importance, being determined by the conditions that it must be wide enough to avoid confusion, and yet not so wide as to hamper the quickness of action of the finger-tips by necessitating excessive stretching. Usually it is just a trifle less than the thickness of a single bead. The same remark applies to the amplitude of the movement of a *pentaroid*, which is usually somewhat smaller than that of a bead in the lower compartment.

How addition and subtraction are done on the *soroban*, would hardly require any explanation. One peculiarity, if it can be looked upon as such, is that operations are performed from left to right instead of from right to left as in the case of written work. This is in perfect harmony with the usual practice of adding or subtracting while numbers are being read off. Thus, for example, in adding 735, as soon as the operator hears "seven hundred," he adds 7 on the *hundreds* rods; meanwhile he hears "thirty" and adds 3 on the *tens;* and hearing "five" adds 5 on the *units*. Here the advantage of the *soroban* is manifest. It remains only to consider addition or subtraction of a digit on a rod. A digit is added to or subtracted from the rod, so to speak, rather than, to or from the digit on the rod. A glance shows whether the operation can be done on this particular rod alone, or the next rod to the left is to be called into play. If the operation can be effected on one rod, it requires no further explanation; in the other case, we need only know the complementary digit, that is, the digit which with the given digit makes 10. For instance,

suppose we have to add 7 on a rod where there is already a digit greater than 2, then we call or think "seven plus three is ten," and add 1 to the next rod on the left and subtract 3 from the rod in question. Again, if we were to subtract 4 from a rod, where there is a digit smaller than 4, then, following the sound of "four away six remains," we subtract 1 from the next rod on the left and add 6 to the rod under consideration. Thus, these operations, even in the most unfavourable cases, involve no mental labour beyond that of remembering the complementary digit. Moreover such phrases as "seven plus three is ten" or "four away six remains" are more condensed and euphonic in their original Japanese than in their English translations; and after a little practice they become so familiar to the operator that they appeal only to what we may call mechanical memory requiring no real mental exertion. He utters a certain muttering sound audibly or mutely, and his finger-tips execute a mechanical movement which accompanies the sound as naturally as the fingers of a pianist yield to the dictates of a musical note. In every respect, the mental labour is reduced to a minimum.

One point more I should like to notice, before quitting the subject of addition and subtraction on the *soroban*. Foreigners and also our own countrymen who have been solely accustomed to written work, in first experimenting with a *soroban*, are liable to mix up a little mental arithmetic, and, by so doing, upset the whole, as a little reflection on their own part would convince themselves. They must make efforts to get rid of their accustomed habit and try to act as mechanically as possible on a *soroban*. If they succeed in so doing, then and only then, they will find addition and subtraction on the *soroban* both more rapid and more certain, because it requires less mental exertion than in written work. As Dr. Knott observes, there is one seeming disadvantage in the *soroban*. It is that the result alone of each step appears in the course of any operation, so that if an error is made, the whole operation

must be repeated over again. This is very often put forward as a serious disadvantage. Indeed this is the opinion of many who go over the question superficially and lack the seriousness required in dealing with a thing which, in spite of its appearance, is, in reality, entirely novel. After all, is it not the final result that we are after? And if an error has been made, repetition is inevitable with written work just as with beads-calculation. The average position of an accidental error lies of course half-way; and this would tell in favour of written work. But on the other hand, beads-calculation is as a rule far quicker than figuring and less liable to error. Moreover, in the case of addition and subtraction on the *soroban*, there is, as was said before, every facility for the operation being done in duplicate simultaneously. These considerations, coupled with the evident truth that in any case verification is necessary, would show that the preceeding conclusion, as to the disadvantage of beads-calculation, is at least premature, if not untenable.

MULTIPLICATION.

Multiplication on the *soroban* differs but slightly from the ordinary written process, being done by means of a Multiplication-table, which is common to both. Our multiplication-table has two characteristic peculiarities, which it may be worth while to notice. The first is the entire absence of a word meaning "times," so that the two factors and their product are called in unbroken succession; thus, for example, instead of "three *times* four are twelve," we say simply "three-four-twelve." The second is that we do not distinguish the multiplier and the multiplicand, but consider them both simply as factors, and always put the smaller number first, if they are different; thus, for example, we never say "four-three-twelve" but always "three-four-twelve," irrespective of whether we are multipying 3 by 4, or 4 by 3. Moreover for the sake of euphony, in calling aloud the three numbers in succes-

sion, sometimes, the ending of the first or the beginning of the second is slightly modified by assimilation, and sometimes the second is linked to the third by a hyphenic monosyllable; thus any incongruity which may occur in calling aloud is eliminated as far as possible, and as the result, we have a compact whole which sounds very smoothy. The modification, just spoken of, is analogous to changing *n* into *m* in compounding a word ending in *n* with another beginning with *p* or, again, it may recall saying *twenty* instead of two tens, *tuppens-hapeni* instead of two pence and half penny, *thrpens* instead of three pence, *happ'orth* instead of halfpenny worth, and the like. The same remark applies to the division table which will be explained later and which is by far the most ingenious contrivance in calculation on the *soroban*. The English multiplication table goes up to 12, and it is not seldom recommended to go on higher up to 16, or, so far as the multiplier only is concerned, up to 19. This seems to be due, at least partly, to the necessity of pushing on mental work as far as possible, in absence of such a convenient means as our *soroban*. Knowing a multiplication table for numbers greater than 9 is not only superfluous but harmful to beads-calculation. Our multiplication table arranged on a square, divided into 81 smaller squares in the manner of a chess-board, would contain only 45 entries; the first row and the first column, each containing digits in order, the diagonal joining 9 with 9 and the part above it only are filled up. Thus the number of mnemonic phrases for multiplication to be learnt by heart is reduced to a minimum, and each of them made as short and euphonic as possible, and so every facility is given to committing them to memory. It is only while practicing, that they must be called aloud; once firmed fixed in the memory, they may be uttered mentally instead of vocally. Doubts may be entertained as to the expediency of interchanging the multiplier with the multiplicand in a multiplication table, when the former is greater than the latter, but experience shows

that there is not even the slightest inconvenience. To this point, however, we shall return later, when treating of written multiplication.

In multiplying two numbers on the *soroban*, in the order from left to right, the multiplier is set first, then a little apart, the multiplicand. On the right of the multiplicand, there must be a number of unused rods not less than the number of places of the multiplier. The operation is essentially the same as in written multiplication, the only difference being that here each figure of the multiplicand is multiplied by the entire multiplier instead of *vice versa*. Again, as soon as a figure of the multiplicand has been used, it is eliminated so that finally the multiplier and the product only remain visible on the instrument.

As an example, let us multiply 6283 by 749. Set these two numbers on a *soroban* in the manner just explained. Observe that here we must have at least three unused rods to the right of the multiplicand. Henceforth, in the diagrams, only the beads which are being used, will be represented. The beads belonging to the multiplicand are given in black, while all others are indicated by outlines. The following Fig. 2. represents the two numbers set on a *soroban*.

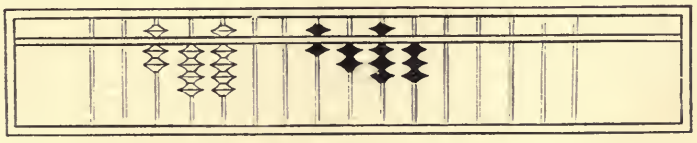

Fig. 2.

Count to the right of the multiplicand as many rods as there are figures in the multiplier; the last one, that is, the third in this case is to be taken for the *units* rod of the product. Multiply the unit figure 3 of the multiplicand by the multiplier 749—in the actual operation 749 by 3. Thus multiply 9 by 3 and set 27; then multiply 4 by 3 and set 12,

the last figure 2 on the *tens* rod where there is already 2 coming from below, making 4 in all; and then multiply 7 by 3 and set 21, the last figure 1 on the *hundreds* rod where there is already 1 coming from below, making 2 in all, and, to make room for setting the first figure 2, the *thousands* rod is to be cleared of the 3 of the multiplicand. The result so far is shown in Fig. 3.

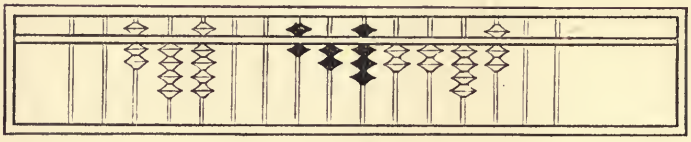

Fig. 3.

Now multiply the next figure of the multiplicand, namely 8 by the multiplier 749. Thus $8 \times 9 = 72$, of which 2 is to be added to the *tens* rod and 7 to the *hundreds* rod; $4 \times 8 = 32$, of which 2 is to be added to the *hundreds* rod and 3 to the *thousands*; $7 \times 8 = 56$, of which 6 is to be added to the *thousands* rod and 5 to the next rod on the left after having cleared it of the 8 belonging to the multiplicand. The result of the process thus far is indicated in Fig. 4.

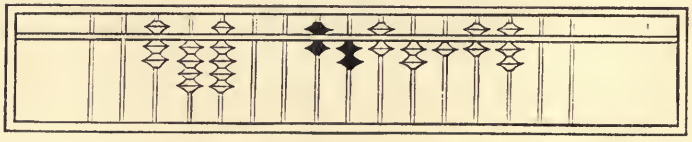

Fig. 4.

The operations with 2 and 6 are carried out in a similar manner, wherein care must be taken to add each partial product on its proper rod, and to remove the figure of the multiplicand which has been just used. The final result giving 4705967 for the required product is shown in Fig. 5.

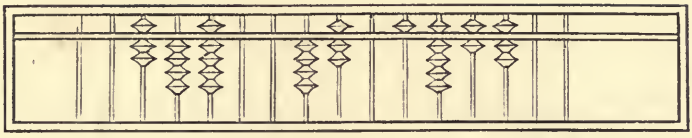

Fig. 5.

Throughout the whole process, every partial product is added as soon as it is named; and thus mental labour is, as in all other respects, reduced to a minimum. With moderate practice, multiplication on the *soroban* will fairly cope with any skilled figuring work.

DIVISION

Division on the *soroban* is essentially the same as the ordinary written process; but it has some peculiarities, which may not be altogether uninteresting.

Let us begin with division by one of the numbers 2, 3, ..., 9. It is done by means of what we may call a Division Table. Set the divisor to the left on a *soroban*, and a little apart to the right the dividend. As the operation goes on, the figures of the dividend gradually disappear from left to right and in their places emerge the figures of the quotient in the same order, so that finally only the divisor and the quotient, and the remainder, if there be any, are left on the instrument. It is important to keep in mind that the denomination of any one particular rod is one place lower for the quotient than for the dividend; thus the *hundreds* rod of the dividend is at the same time the *tens* rod of the quotient. An example will make all this clear. Let it be required to divide 385 by 2. In the following figures the beads belonging to the dividend are given in black, while all others are represented by outlines.

In the following figure, A represents the divisor and the dividend set on a *soroban*. Now the division table for TWO contains two entries; they are "two one replace by

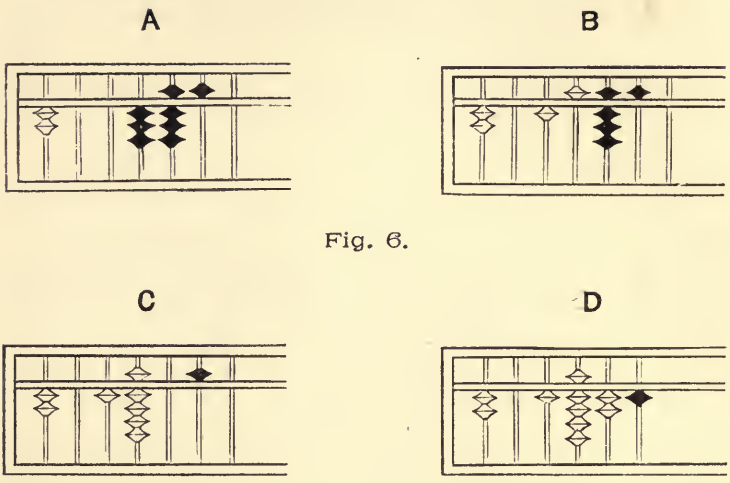

Fig. 6.

five," and "two two up one" Firstly, 3 on the *hundreds* rod of the dividend is to be operated upon; calling "two two up one," take off 2 therefrom and put 1 on the next rod to the left; then calling "two one replace by five," replace the remaining 1 by 5. B shows the operation so far. Secondly, operate on the 8 of the *tens* rod of the dividend; repeat four times "two two up one," every time taking off 2 and adding 1 to the next rod on the left. In passing, we may notice that we might just as well have said "two eight up four," and reduced the whole to a single operation. C shows the process thus far. Thirdly, operate on the 5 of the *units* rod of the dividend; repeat twice "two two up one," every time taking off 2 and adding 1 to the next rod on the left; there is still left 1 which is the remainder. Here also, we might have said "two four up two" and have done the whole process in a single stroke. D shows the final result with 192 for the quotient and 1 for the remainder.

We now give the Division Table consisting of eight

minor tables for the divisors 2, 3, 4, ... , 9 respectively. In translating, no effort has been spared to make it as literal as possible; but, wherever it was thought likely to prevent misinterpretation or conducive to better understanding, I have allowed myself to deviate slightly from the original. For all that, the table given below would be hardly intelligible without explanation; and this is not to be wondered at, as its sole purpose is to serve for a mnemonic.

Division Table for TWO

two one replace by five	two two up one

This table may be supplemented by the following:

two four up two „ six up three	two eight up four

Division Table for THREE

three one three down one „ two six down two	three three up one

This table may be supplemented by the following:

three six up two	three nine up three

Division Table for FOUR

four one two down two „ two replace by five	four three seven down two „ four up one

This table may be supplemented by "four eight up two."

Division Table for FIVE

five one add one „ two add two „ three add three	five four add four „ five up one

Division Table for SIX

six one down add four „ two three down two „ three replace by five	six four six down four „ five eight down two „ six up one

Division Table for SEVEN

seven one down add three	seven five seven down one
„ two down add six	„ six eight down four
„ three four down two	„ seven up one
„ four five down five	

Division Table for EIGHT

eight one down add two	eight five six down two
„ two down add four	„ six seven down four
„ three down add six	„ seven eight down six
„ four replace by five	„ eight up one

Division Table for NINE

nine one down add one	nine six down add six
„ two down add two	„ seven down add seven
„ three down add three	„ eight down add eight
„ four down add four	„ nine up one
„ five down add five	

As an example for illustrating the use of the table for SEVEN and at the same time explaining its meaning, let us divide 1285 by 7. On a *soroban* set the divisor 7 to the left and a little apart the dividend 1285, as shown in the diagram on the left of Fig. 7.

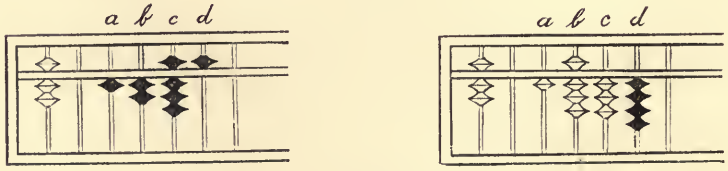

Fig. 7.

As said before, rod (a) is the *thousands* for the dividend and at the same time the *hundreds* for the quotient; a

similar relation holds with respect to rods (b) and (c); and finally rod (d) is the *units* for the dividend and at the same time the place for showing the remainder. Now begin with rod (a): the table says "seven one down add three," which means, leave 1 as it stands and add 3 to the next rod (b); by so doing, this 1 is transmuted from 1 *thousand* of the dividend into 1 *hundred* of the quotient. Secondly, there is now 5 on rod (b): the table says "seven five seven down one," which means, change 5 into 7 on rod (b) and add 1 to the next rod (c). Thirdly, there is now 9 on rod (c): the table says "seven seven up one," which means, subtract 7 from rod (c) and add 1 to the next higher rod (b); with respect to the 2 remaining on rod (c), the table says "seven two down add six" which means, leave 2 as it stands and add 6 to the next rod (d). Fourthly, concerning rod (d), on which there is already 5, we had to add 6 as the result of the last step; but this can not be done as is evident at a glance. So, calling "seven seven up one," add 1 to the next higher rod (c) and subtract 1 (the difference between the divisor 7 and the wandering 6) from rod (d) instead of adding 6 to the same rod. Observe that this is equivalent to taking off 7 from the rod overfilled with 11 and adding 1 to the next higher rod. Lastly, on rod (d), there remains 4 which is left untouched as the remainder. The final result with the quotient 183 and the remainder 4 is shown in the diagram on the right of Fig. 7.

Let us now notice some of the salient features of the process of division by a digit on the *soroban*, as exemplified above. First of all, it can hardly escape notice that it is more like *long division* than *short division*. Again, in dividing by 7, obviously 7 of the dividend is equivalent to 1 of the quotient, whatever its denomination may be, and the one can be replaced by the other at pleasure. Of this principle free use is made; and as the result, the number

to be really dealt with on a particular rod is always less than 7. The culminating feature which dominates the whole, lies in decomposing the entire process into as many steps as possible, so that, in each step, the operator is only concerned with a single rod and every thing can be done by mechanically following the division table.

The division table for SEVEN having been already explained, there will be no difficulty in understanding the meaning of other tables. The various ways of expressing the same thing, such as "add 1" instead of "replace 2 by 3," are taken advantage of, in making the phrases as curt and melodious as possible. This accounts for the non-uniformity of expressions in these tables. Among them, there are two which are particularly laconic in their expressions and fitting in their application, namely the one for FIVE and the one for NINE. Readers who might happen to feel interest in what has been just said, are recommended to make trials of using these tables in dividing any number they may think of by 5 or 9. They would soon perceive that division by 5 simply amounts to multiplying by 2 and dividing by 10, and division by 9 is nothing but the nearest approach to the inverse operation of that method of multiplication by 9 which consists in multiplying by 10 and subtracting the dividend; but with the *soroban* the processes are singularly elegant and peculiarly rapid.

It remains for us to consider *long division*. Here, in addition to the tables already given, we make use of the following table commonly known by the name of "See-One."

See-One Table

see one no head nine down one	back one in one
„ two „ „ „ „ two	„ „ „ two
„ three „ „ „ „ three	„ „ „ three
.....	.....
see nine no head nine down nine	back one in nine

To explain the meaning of the above table, as an example, consider the case of dividing by a number whose first figure is 7. Suppose the first figure of a partial dividend is also 7. Now in order to get a trial figure of the quotient, we have to divide the first figure of a partial dividend by the first figure of the divisor. Were we to follow "seven seven up one," we would have to put 1 on the rod next higher to the rod of the dividend under consideration. But this 1 is very likely to be too great; so instead of "up 1" which is equivalent to 10 on the rod in question, we put 9 on it and 7 on the next lower rod which still belongs to the dividend. This is expressed by "see seven no head nine down seven." Again, "back one in seven," which may be looked upon as an abbreviation of "give back one and take in seven," means take off 1 from a trial figure of the quotient and restore its equivalent 7 to the dividend.

In *long division* just as in *short division*, as the operation goes on, the dividend gradually disappears, so that finally only the divisor, the quotient, and the remainder are left on the *soroban*. For the clear understanding of the actual process, it would be difficult to overestimate the importance of the following principle. A rod serving first for the dividend and then for the quotient, has, for the quotient, a denomination lower than for the dividend, by the number of figures of the divisor. Thus, for instance, in dividing by a number consisting of 3 figures, the *ten-thousands* rod of the dividend is at the same time the *tens* for the quotient.

Let it be required to divide 703,314 by 738. The same example is worked out in the paper of Dr. Knott already referred to. I am going to give an explanation of the process somewhat different from that of my friend. In passing, it may be observed that he does not explicitly mention the "See-One" table just spoken of. There is al-

ways something to be learned, or even a peculiar charm, in comparing the explanations of one and the same thing from different points of view. It is for the benefit of those who may have access to Dr. Knott's paper, that I have purposely chosen the same numbers.

The ordinary written process of dividing 703314 by 738 way be divided into the following three stages :—

I. The 1st partial dividend 7033 divided by 738 gives 9 for the 1st figure of the quotient; $738 \times 9 = 6642$; subtracting this from 7033, there remains 391.

II. The 2nd partial dividend 3911 divided by 738 gives 5 for the 2nd figure of the quotient; $738 \times 5 = 3690$; subtracting this from 3911, there remains 221.

III. The 3rd partial dividend 2214 divided by 738 gives 3 for the 3rd figure of the quotient; $738 \times 3 = 2214$; subtracting this from 2214, there remains nothing.

Now, on the *soroban*, the above three stages are each subdivided into a number of steps, so that in each step the operation can be effected by mechanically following the division table. Fig. 8. represents a *soroban* with the divisor 738 in outlines and the dividend 703314 in black set on it in proper position.

Fig. 8.

First of all observe that rod (a) is the *hundreds-thousands* for the dividend and the *hundreds* for the quotient. Again, bear in mind that in each stage the start is made by considering the first figure 7 of the divisor.

Now the 1st stage is subdivided into the following 3 steps :—

(1) Operating on rod (a), say "see seven no head nine down seven." On rod (a) replace 7 by 9; this 9 belongs to the quotient. On rod (b) put 7; this 7 belongs to the dividend.

(2) Multiply the second figure 3 of the divisor by 9; $3 \times 9 = 27$. Subtract 2 from rod (b) and 7 from rod (c). As the latter subtraction can not be effected, subtract 1 more from rod (b) and add 3, the complement of 7, to rod (c).

(3) Multiply the third figure 8 of the divisor by 9; $8 \times 9 = 72$. Subtract 7 from rod (c) and 2 from rod (d). As the former subtraction can not be done, subtract 1 more from rod (b) and add 3, the complement of 7, to rod (c). This completes the 1st stage. The result so far is shown in Fig. 9.

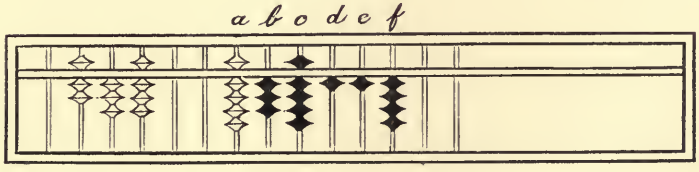

Fig. 9

Next, the 2nd stage is subdivided into the following 4 steps :—

(1) Operating on rod (b) which now reads 3, say "seven three four down two." On rod (b) replace 3 by 4; this 4 belongs to the quotient. It remains to add 2 to rod (c) which has already 9; but as this can not be done, pass on to the next step.

(2) Make use of "seven seven up one," and take off 7 from rod (c) and put 1 to rod (b); this 1 belongs to the

quotient, so that the second figure of the quotient now becomes 4+1 i.e. 5. To rod (c) add 2 which was to be added at the end of the last step.

(3) $3\times5=15$. Subtract 1 from rod (c) and 5 from rod (d). As the latter subtraction can not be done, subtract 1 more from rod (c) and add 5, the complement of 5, to rod (d).

(4) $8\times5=40$. Subtract 4 from rod (d). This completes the 2nd stage. The result of the operation so far is shown in Fig. 10.

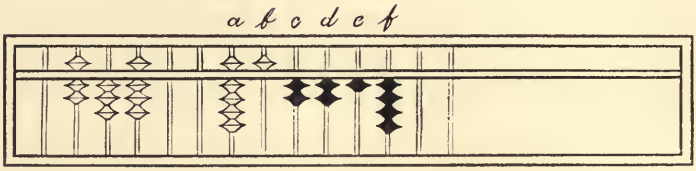

Fig. 10.

Next, the 3rd stage is subdivided into the following 4 steps :—

(1) Operating on rod (c) which now reads 2, say "seven two down add six." Leave 2 on rod (c) untouched. Only notice this 2 now belongs to the quotient. Add 6 to rod (d).

(2) On rod (d), there is now 8. Say "seven seven up one," and take off 7 from rod (d) and add 1 to rod (c). This 1 belongs to the quotient, so that the third figure of the quotient now becomes 2+1 i.e. 3.

(3) $3\times3=9$. Subtract 1 from rod (d) and add 1, the complement of 9, to rod (e).

(4) $8\times3=24$. Subtract 2 from rod (e) and 4 from rod (d).

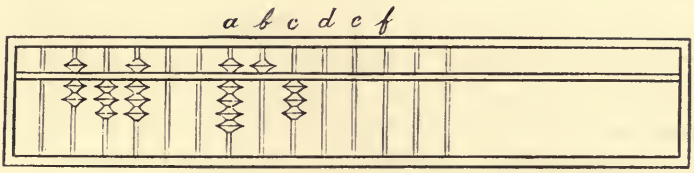

Fig. 11.

Thus the whole operation ends, there being no remainder. As represented in Fig. 11., on rod (a) there is 9, on rod (b) 5, on rod (c) 3; all taken together, they represent the quotient 953. Were the dividend 703314 plus some number less than the divisor 738 instead of 703314, then this number would appear as the remainder on rods (d), (e), and (f), which are now empty.

In the foregoing, we have not as yet exemplified the use of that part of the "See-One" table which runs "back one in a digit." This is to be brought into play, when a trial quotient had been assumed too great. For instance, if, in the example worked out above, the second figure of the divisor were 9 instead of 3, we meet with the impossibility of subtracting 81 from 73 at step (2) of stage I; so we have to make use of "back one in seven," and take off 1 from rod (a) and add 7 to rod (b). Now, although we can not add 7 to rod (b) which has already 7, forthwith we have to subtract the first figure 7 of the product of 9 and 8; so, after all, there is no real difficulty.

Here in division, as elsewhere, the practical merit of the *soroban* lies in the Spartan husbanding of brain work. Indeed, mental labour is economised to such a degree of perfection, that it is by no means an idle talk or an exaggeration to speak of its complete absence. The marvellous potency of the *soroban* is, in the main, due to the conventional division table, which, to my great regret, has lost through translation much of the ideal beauty of the original.

It is to be hoped that an abler hand would some day recast the version into a nearer approach to the original.

Extraction of square root and cube root can also be done on the *soroban*, aided by appropriate tables. The actual processes, though not essentially different in principle from the ordinary written method, are, in many respects, ingenious and even fascinating. But their elucidation lies beyond the scope of this report, and is reserved for some future opportunity.

There is one thing which I might have mentioned before, but which I have purposely kept back till I have nearly done with the beads-calculation. It is this, that sometimes a *soroban* has two pentaroids instead of one. On such an instrument, we can set on a single rod any number not exceeding 15, and this is rather convenient in division, though of course not necessary. Thus in the example of division worked out previously, in stage I, we could have added 2 at the end of step (1) without waiting till the end of the next step. However, the existence of two pentaroids destroys the indispensable simplicity of the *soroban*. We would have made a poor bargain, were we to obtain a trivial convenience at the expense of this simplicity. Those who advocate the use of a *soroban* with two pentaroids, are to be classed among those who have only one eye in looking into such matters.

In olden days prior to the birth of New Japan, students of beads-calculation were told to commit tables to memory and to do so and so, without being told why they must do so. They were taught dogmatically, practiced it blindly and applied it mechanically to the exigencies of daily life. Thus from beginning to end it was an involuntary procedure. Books giving information on the subject were also written in the same spirit. Only a few gifted ones who are capable of learning without being taught, saw, as it were from a distance, like looking through a mist gradually

disappearing before the rising sun, the principle which underlies the operations; but even they were unable to explain it to others.

On the awakening of Regenerated Japan, this negligence on the part of those who taught the art of calculation on the *soroban*, was destined to incur a severe penalty in the form of a bitter attack for the purpose of extirpating it altogether. It was argued, everything to be taught in the new schools must contribute more or less to mental culture, the art of calculation on the *soroban* is purely mechanical, and therefore it must be altogether excluded. And this argument, primitive as it is, was, at that time, considered to lie beyond the danger of its validity being even questioned, in the tumultuous atmosphere whirled about by a hurricane sweeping away everything old to make space for something new.

The period of chaotic transition now being over, let us calmly consider the question on its merits. In actual practice the soroban-calculation is as mechanical as it can possibly be, and indeed, as repeatedly asserted, therein lies its advantage as *versus* written calculation. But in teaching it, there is no reason whatever, why we should not explain the principle on which the operations are based. Only by so doing, the learner can get an intelligent grasp of the processes, and thus he would learn more quickly than otherwise. That, after sufficient practice, his finger-tips will move mechanically is no fault of this method of teaching. Does not a similar thing occur in the case of written calculation? After some practice, in written work too, are not the calculations performed more or less mechanically? Only not so mechanically as in the case of calculation on the *soroban*. To come to the point, beads-calculation, if properly taught, would contribute to intellectual culture at least as much as the ordinary written arithmetic. Numbers are to be analysed into such constituent parts as befits the end in view and

various ways of combining these parts must be discussed in order to give an adequate explanation of the processes. The explanation would be easy to comprehend, because any one process is decomposed into a number of so many independent steps and each particular step need be cleared up at a time. It may be observed in passing, that this is in perfect harmony with the so-called Socratic method of teaching so successfully applied to the secondary education of the country standing prominently in the front rank of educational progress. Meanwhile intellectual training is going on, the pupil becoming gradually familiar with numbers and acquiring little by little instinctive insight into the various processes of calculation.

As concerning the great mass of people, the teaching of arithmetic is by far the most important thing within the range of mathematical instruction. Now, beads-calculation having asserted itself as something indispensable to our national life, we find ourselves confronted with the problem of problems: How is it to be taught in our elementary schools? There is no ground whatever why it should remain a separate entity apart from the ordinary arithmetic. For, when all is said, is it not simply a variety of the processes of calculation? If so, is it not eminently proper that it should form a part of our arithmetic? Again, there is always a limit to the number of hours allotted to the teaching of arithmetic in general education, and the including of beads-calculation ought not to give rise to complaints as to insufficiency of the time at disposal. So the question comes to this: How are we to incorporate beads-calculation within our arithmetic, without bulging it out abnormally?

The teaching of arithmetic covers the whole six years of elementary education and stretches out more than a year into middle school education. Whewell, the author of the well-known "History of Inductive Science," who possessed

the double capacity of being a senior wrangler and at the same time one of the most profound thinkers of his days, said some fifty years ago in his essay on "Liberal Education": "Arithmetic is a matter of habit, and can be learnt only by long-continued practice. For some years of boyhood there ought to be a daily appropriation of time to this subject." Had he been alive to-day, he would have insisted upon this point even more vigorously. Now in any work requiring pratice, the more pratice the better. But here we have to take into account human weakness which manifests itself in becoming tired of monotony even when engaged in training for a work requiring practice, which means the repetition of the same thing over and over again, and is therefore necessarily monotonous. As we are pursuing this trend of thought, the idea may suggest itself: After all beads-calculation may furnish just what is wanted in arithmetical practice, namely, variety in uniformity otherwise called monotony — variety, because the processes are apparently somewhat different from the ordinary methods, and uniformity, because they are only slight modifications of operations in reality equivalent. The problem formulated above has not as yet been solved; but there is a fair chance of its being solved satisfactorily, and, more than that, it may be capable of a very happy solution. Here is a task which awaits the best efforts of any one who is broad-minded and capable of considering such matters from every point of view.

Nowadays we hear so often of the reform movements in the teaching of geometry, algebra and other higher branches of mathematics. It seems rather odd that we hear so little of the reform movements in the teaching of arithmetic. Has it reached such a state of perfection that nothing remains to be desired? Or perchance, is it that such a question lies below the dignity of such brains as those now busily occupied with the reforms in the teaching of the higher

branches of mathematics? The last odd thirty years have witnessed wonderful changes all over the world. The intricacies of modern civilization very often, if not invariably, find their exponents in the form of numerical expressions. It is hard to suppress a vague idea that something might be done to the art of everyday calculation to keep pace with the surrounding changes, and, as a natural consequence as well as on its own account, something might also be done to the teaching of arithmetic. If this idea is not to be despised, then soroban-calculation may have a claim to consideration in a circle wider than our own.

Before closing this chapter, one thing more remains to be said concerning multiplication and division on the *soroban*. In multiplication, we may evidently just as well begin the operation with the highest figure of the multiplicand instead of the lowest. The process is then called "From-head-downwards" multiplication. Again, in division, there is a slightly different method, usually known by the name of *Kameizan*, which is more nearly allied to the ordinary written process than the one exposed in the above. All these have, however, been passed over, so as not to confuse those readers, to whom beads-calculation may be entirely new.

CHAPTER III

THE TEACHING OF ARITHMETIC IN ELEMENTARY SCHOOLS (*continued*)

The somewhat great length, to which the account of soroban-calculation in the preceding chapter has swelled out, ought not to convey an erroneous impression, as if it stands on the level with written calculation. It is the written calculation which reigns supreme in our elementary education; calculation on the *soroban* is associated with it only as an indispensable accessory. In the present chapter we shall be chiefly concerned with the teaching of arithmetic in elementary schools.

The object of elementary education, as defined in the official text, is as follows :—

"In elementary schools, children are given the rudiments of moral and civil education, together with such practical knowledge and ability as are necessary in ordinary life, due regard being had throughout to their gradual bodily development."

The regulations, now in force, regarding the various subjects to be taught in the elementary school, give the following directions as regards the teaching of arithmetic :—

"The main object to be aimed at in the teaching of arithmetic is to give children proficiency in simple calculation of everyday need, to impart them a varied knowledge connected herewith and necessary in daily life, and at the same time to habituate them to be clear and exact in thinking. In the elementary school, numeration and notation,

addition, subtraction, multiplication, and division are to be taught, at first within the domain of numbers not greater than 10, and afterwards enlarging this domain to numbers not exceeding 100. Then the four operations in any integers, decimals, compound numbers, simple fractions, and percentage are to be taught. In the higher elementary school, some more of fractions and percentage are to be taught; and if the length of the course allows, mensuration is to be taught, and according to local conditions, soroban-calculation may also be taught in addition. In teaching arithmetic, children must be made to understand clearly and accurately, and to become experienced in calculation, so as to be able to apply it practically and freely; they should also be able to explain the method and rationale of calculations. They are also to be made proficient in mental calculation. Examples in arithmetic should be chosen with reference to what have been taught in other subjects, and to the local circumstances, and should be such as are in close touch with the exigencies of daily occurrence."

The reader who has gone through the last chapter, will hardly fail to notice in the above directions some remnants of the prejudice against beads-calculation, prevalent at the time when elementary schools were first started. Nothing further is said than, that according to local conditions soroban-calculation may also be taught in addition. In conformity with the spirit of looking upon soroban-calculation with something like scorn, this statement was often interpreted to imply that soroban-calculation must not be taught unless necessitated by special local circumstances.

Now, in view of the almost excessive centralization of authority in educational matters spoken of in the first chapter, it is but natural that whatsoever directions are issued by the Department of Education should be followed to the letter. Sometimes, this tendency went even further. Prescriptions

couched in the words "may be" were not seldom interpreted in the sense of "must be," as if this course of action were more in harmony with the dispositions of the all-powerful central authority. Under these circumstances, it is no wonder that in our elementary education soroban-calculation should have received, to use a colloquial Japanese expression, the treatment of a step-child. Thus it has come about that beads-calculation is either not at all or but inadequately taught in elementary schools. The use of the *soroban* among the people at large, however, has not ceased in the least. On the contrary, ever since the birth of Rejuvenated Japan, the need of soroban-calculation has, so to speak, grown up, like so many other things which are really useful, no one quite knows why or how, as though it were native to the soil. And since soroban-calculation had the misfortune to be slighted in the regular school course, our young people coming from the elementary schools were obliged to learn it somewhere else and somehow under adverse conditions. All over the country, especially in rural districts, one meets with private instructors of soroban-calculation and with small evening classes where beads-calculation alone is taught. Now, to come to the point, I see no reason whatever, why soroban-calculation should not be included among "such practical knowledge and ability as are necessary in ordinary life" spoken of in defining the object of elementary education.

It is now our purpose to give some details of the teaching of arithmetic in elementary schools. Before doing so, however, we may be allowed to say a few words in general concerning the text-books used in these schools. Nowadays, all over the world, we often hear of the superfluity of books, and not seldom do we meet in this connection with such strong words as "ever-increasing haystack of rubbish," "cemetery of dead and superannuated books," and the like. Now there can never be a superfluity of really good books,

and a truly valuable book never becomes superannuated. As the proverb says "where there is smoke there is fire," the existence of such complaints may be looked upon as a proof that there are books of questionable value and that it would have been better for the world if they had not been published. Unfortunately for the cause of education, what has been just said applies particularly, and but too much so, to text-books.

A few years ago I prepared a list of books which are usually designated by such names as "Calculus for Beginners," "Introduction to Calculus," "First Course in Calculus," and the like, and which were either newly published or re-edited within the last ten years or so. My list, which was printed for private circulation, was limited to works written in English. Moreover, I have listed only those which happened to come to my knowledge without making any special efforts to be exhaustive. Indeed, there is every reason to believe that the list is very far from being complete. And yet it contains more than 100 entries! Had I not confined myself to English books and, moreover, tried to leave no stone unturned, I can hardly imagine the extent, to which my list would have swelled out. In this connection, I can not help recalling a mathematician and author well-known by his admirable works on the theory of functions. Even he apologizes for publishing a new text-book of elementary calculus. In the very first line of the preface, we meet with the exclamation: "*Noch ein Lehrbuch der Differential-und Integralrechnung?! War denn das nötig?* And then he tries to justify himself, perchance not without difficulty, for making an addition to the already long list of German text-books on elementary calculus. That even a mathematician of his fame should feel so shy in writing a new text-book on the subject and apologize so much for publishing it, seems to bespeak the superabundance of such books already in existence.

What has been just said, has, of course, nothing to do with the teaching of arithmetic in elementary schools. It is merely to serve as an example illustrating the superfluity of text-books. Now the case of school text-books seems in all probability to be by far worse than that of elementary calculus. The golden days when every author wrote a book because he was perfectly sure of producing something far better than any thing he knew of and was fully conscious of having sacrificed his time and labour for the benefit of others, are long since past. Nowadays, authors are frequently at the mercy of booksellers and publishers. Sometimes they write books, although they know beforehand that their own production will not after all materially differ from those already in existence. In some such ways, text-books multiply. No wonder that we hear of their superfluity.

Now the superfluity of text-books, taken by itself, is not a positive evil. Only it is liable to give rise to competition. Even competition is not an evil; on the contrary, if confined within the limits of really improving the contents of books, it is something very much to be desired. However, it is not to be wondered at that publishers are but too eager to see their publications adopted for class use. But when they push their efforts in this direction too energetically, wherein human weakness is often liable to manifest itself in manners and ways quite unlooked for, then competition begins to assume a dangerous character, similar to the preaching of war for the sake of ship-building contracts, and ends in a corruption like that of Tammany Hall so often heard of in the world of politics and municipal administration, and which ought to be absolutely excluded from matters pertaining to education. The moral influence, though indirect, of such corruption upon the younger generation is simply disastrous.

It requires courage to say openly what many educa-

tionalists have long had in their mind and sometimes whisper to one another in private. I myself would gladly pass this over in silence; but for the sake of the noble cause of education let the facts be told plainly. Now, whatever may be the avowed purpose of a publisher in publishing a text-book, surely it is of the utmost importance to him that the book should sell. There may be noble exceptions; but in the majority of cases, it is but natural that, after all, no other consideration is of more weight than the sale of the book. Often in the advertisements of a newly published school text-book, we meet with announcements like the following: "Specimen copies will be sent on application pending the eventual adoption for class use." Now sometimes it happens that some one asks for a specimen copy for no other purpose than getting it gratuitously for himself. Publishers become more cautious and say something like the following: "Applications from teachers for specimen copies will be considered. It is requested that the number of copies likely to be purchased for class use, if the book is adopted, be stated with the application." Now if the matter goes only so far and stops at that point, there is nothing to be said against the practice. But if it goes further, and the efforts of publishers in inducing teachers to adopt particular text-books for class use are pushed on too vigorously, then it becomes at once a catastrophic evil. Competition as a consequence of overproduction is in all cases prone to leaping over the bounds of fair dealing dictated by business integrity and even by conscience. In the present case, as stated before, competition is to be welcomed, provided it takes the course of bettering the contents of text-books, using better kind of paper, improving the printing and binding, and the like; but once it takes the form of putting teachers, or whosoever has a voice in the adoption of text-books for class use, under obligations to the publishers and, as a consequence, under

pressure impeding their free choice, then, it is by no means an exaggeration to say that competition becomes a catastrophic evil. Nothing can be more harmful to the noble cause of education than putting in doubt the integrity and conscience of those who come in daily contact with the younger generation and are held up as models worthy of imitation.

Here in Japan, we have had a very sad experience as regards elementary school text-books. Under the unification due to the almost absolute centralization of authority in educational matters, repeatedly spoken of, the evil seems to be capable of assuming, and actually did assume, so gigantic a magnitude that it led to the famous "text-book scandal" of 1903. The details of this ignominious affair are too painful to be rehearsed here. However, the evil seems to exist more or less elsewhere too, only not to such an extent as here in Japan previous to 1903, and the story of this scandal can not possibly fail to be instructive to those who may wish to see what this world-wide evil looks like when magnified upon a gigantic scale. They are referred to a tolerably complete account of this affair to be found in chapter XXIII of Baron Kikuchi's work already cited.

Ever since 1903, we have the so-called "State text-book system." The regulations now in force regarding elementary school text-books are essentially as follows: As a rule, text-books to be used in elementary schools must be those whose copyrights belong to the Department of Education. They are books either compiled by the official compilers in the Department, or by private individuals at the request of the Department, but in all cases copyrighted by the Department, so that no private persons shall have any interest in them from a business point of view. However, as regards a few special subjects, the Minister of Education may give permission for using, in elementary

schools, text-books whose copyrights do not belong to the Department, but which have been examined and approved of by the Department. Again, with respect to a few subjects like gymnastics, sewing, and manual work, no text-books are to be used.

So far as I am aware of, Japan is unique among the principal countries of the world in having a State text-book system for elementary schools——the system under which, in all the elementary schools throughout the country, the same text-books compiled by the central authority are used. Indeed, I believe, this is possible only where there is almost absolute centralization of authority in educational matters, and is most probably to be found nowhere else than here in our country. We have now experience of several years of this system. Nearly a decade having elapsed since the abrupt introduction of the system, the time has arrived, when, free from the influence of the tumult caused by the scandal of 1903, we can soberly think over the merits as well as the demerits of the system. In so doing, we can not help becoming more and more convinced of the evident truth that competition within proper limits is necessary to progress in any matter and under all conditions. It is true, that the Department of Education is constantly making efforts to improve the contents of the State text-books. Not only are the official compilers charged with the task of revising them continually so as to improve them and keep them up to date, but, with the same end in view, teachers in normal schools and those actually using the books in their classses are urged to send in any suggestions that may occur to them. Thus no effort is spared for improving the State text-books so far as is compatible with bureaucratic conditions. But it is a pity that bureaucratic conditions are not the conditions under which the best efforts of the fittest can be called into play. The difficulty of compiling a text-book under conditions which

harmonize with bureaucratic traditions, would far surpass the imagination of any one who has had no actual experience. After all, it may lie in the nature of the thing itself, that a State text-book on almost any subject should come out as dry as a legal document. It lacks that something which stimulates the mind of the children and arouses their desire to learn, and will result, in the course of time, in their becoming fond of the subject.

So far, we have been speaking of the State text-books in general. Let us now consider, in particular, the State text-books on arithmetic. It is only natural that the compilers should be very cautious or even over-cautious as to their production being unassailable from any and all points of view. If there could be found any thing in their compilation, against which something might be said, even though it be not a positive fault or mistake, they would feel it as a slur upon the bureaucratic dignity commensurate with the almost absolute power of the central authority. Now there is always some sort of difficulty in any work, and the work of compiling State text-books is no exception to the rule. And wherever there is a difficulty, there always remains a margin for comments as to the manner in which the difficulty might have been overcome. So, they are obliged to have recourse to the simplest of all means of overcoming difficulties either by skipping over i.e. omitting altogether any points which are likely to give rise, if not to a controversy, at least to a disagreement of opinions, or else by offering categorically, without discrimination, various alternatives corresponding to different opinions, the result of which procedure can not possibly fail to be appallingly perplexing. Thus heterogeneous matters are arranged with occasional gaps, and the result lacks that unity and predominating spirit which are essential characteristics of a well-written text-book. It is not matter for surprise——indeed it lies in the very nature of things—that notwithstanding the best

intentions and painstaking efforts of the compilers some of the State text-books should come to light as unattractive as food without taste or as a tumble-down shanty serving as a family domicile. Not the least noteworthy feature is the unification, i.e. the using of practically the same State text-books in all the schools throughout the country notwithstanding the fact that the regulations leave some slight margin in this respect. In view of the difference of local circumstances, this seems to involve a contradiction in itself.

The State text-book system has had its day. It has served a useful purpose in the chaotic period immediately succeeding the "text-book scandal." But now the time seems to have arrived when something must be done to modify the system at present in vogue. Attempts should be made to enable the best efforts of the ablest men in the field to be utilized in the most efficient manner for furthering the progress of elementary education.

That such a thing as the State text-book system is possible, may be looked upon as one of the many symptoms, by which the excessive centralization of authority in educational matters manifests itself. It is not only quite unknown, but would probably be altogether unimaginable in countries like England or America where, as it seems, educational organizations and interests are comparatively free from undue restraint. In this respect I have been always in the habit of considering my own country to be the one extreme and countries like England or America the other. For me, indeed, it was a revelation to hear that, even in England, a man of the wide knowledge and ripe judgement of Sir John Gorst spoke quite recently in a tone like the following: "The time is thus ripe for a drastic revolution in the red-tape methods by which education is tied and bound, and the tyranny of the Board of Education over local authorities could now be relaxed or altogether removed." It appears as if, all over the

world people are here and there complaining of the excessive power vested in a central educational authority. The centralization of authority in educational matters has its advantages as well as its disadvantages, and the twentieth century seems to be confronted, among the many difficult problems to be solved, by the enigma of finding out the best means of somehow minimizing its disadvantages without compromising its advantages.

We now return to the details of the teaching of arithmetic in elementary schools. As the instructions are given by closely following the State text-books on arithmetic, we cannot do better than give an account of them. There are two sets, one for teachers and one for children. Text-books in the set for teachers are called "Teacher's Books," and those in the set for children "Child's Books." There are six volumes of Teacher's Books——one volume for each school year. Children are not to have their own text-books until they have finished their second year, so that there are only four volumes of Child's Books——one for each of the last four school years. Teacher's Books from the third year upwards, have a feature which it may be worth while to notice. Namely, every page is divided into two parts by a vertical line, and on the right of the line is to be found a reproduction from a Child's Book in a smaller print than in the original, while on the left are given explanations, suggestions, answers to problems, additional examples, and so forth, in fact everything which may be of service to teachers in teaching the matters contained in the text on the right of the same page.

We are now going to give the list of contents of Teacher's Books, which will show approximately the order and progress of the teaching of arithmetic in elementary schools. We premise a few remarks which apply to all the volumes. Each volume is divided into three parts corresponding to the three terms of a school year. As to the examples in abstract

numbers, efforts have been made so as to cover all the possible cases; but, as regards practical examples in concrete numbers, merely a few illustrative examples are given and, in addition, the names only of concrete units on the margin of pages, and teachers are required to supply some more and such examples as lie within children's experience. For the sake of conciseness, symbols and abbreviations are freely used throughout, and teachers are insisted upon not to forget to translate them into proper expressions, before making use of them in classroom.

In order to understand the list of contents to be presently given, it is important to notice a peculiar characteristic of our numeration. We have, for numbers up to 10, two ways of counting or rather two sets of names. One set which is most commonly used in actual counting and particularly by small children, may provisionally be called the *primitive* numeration: it runs *hitotsu, futatsu, mittsu, yottsu, itsutsu, muttsu, nanatsu, yattsu, kokonotsu, tō.* The other set may be called numeration simply, or, if necessary for distinguishing it from the preceding one, the *formal* numeration: it reads *ichi, ni, san, shi, go, roku, shichi, hachi, ku, jiu.* Numbers greater than 10 are named in conformity with the latter set, and each of them has one and only one appellation.

We now give the list of contents of Teacher's Books on arithmetic.

First school year——The object of this year's course is to familiarise children with, and give them clear and definite conception of, numbers not greater than 100. Mental calculation within the limits of numbers not exceeding 20, in particular, addition of two digits and its inverse operation of subtraction, are to be taught; and thus the foundation of the general processes of addition and subtraction is to be laid.

I. Addition and Subtraction

The *primitive* numeration——Adding 1 to numbers not exceeding 10 ——Adding 2 to numbers not exceeding 5 ——Adding 3 to numbers not exceeding 5 ——Adding 4 to numbers not exceeding 5 ——Adding 5 to numbers not exceeding 5 ——The *formal* numeration——Adding 2 to numbers not less than 6 ——Adding 3, 4 to numbers not less than 6 ——Adding 6, 7 ——Adding 8, 9 ——The Arabic notation——Comparison of two numbers in relation to their being greater or less—Subtracting 1 ——Subtracting 2 ——Subtracting 3 ——Subtracting 4 ——Subtracting 5 ——Subtracting 6 ——Subtracting 7 ——Subtracting 8, 9 ——Zero or Naught——Ideographs representing numbers from 1 to 10 ——Review (*1*).

II. Addition and Subtraction

Numeration of numbers from 11 to 19 ——Adding 1, 2, 3 ——Adding 4, 5, 6, 7, 8 ——Adding numbers not less than 11 ——Notation of numbers up to 19 ——Comparison of two numbers in relation to their being greater or less——Subtracting 1, 2, 3 ——Subtracting 4, 5, 6, 7, 8 ——Finding the difference of two numbers——Adding such a digit to 9, 8 that the sum comes out not less than 11 ——Adding such a digit to 7, 6 that the sum comes out not less than 11 ——Adding such a digit to 5, 4 that the sum comes out not less than 11 ——Adding such a digit to 3, 2 that the sum comes out not less than 11 ——Subtracting 2, 3 from such a number that a digit remains——Subtracting 4, 5 from such a number that a digit remains——Subtracting 6, 7 from such a number that a digit remains——Subtracting 8, 9 from such a number that a digit remains——Subtracting numbers not less than 11 ——Numeration and notation of 20 and calculation involving it——Review (*2*).

III. Addition, Subtraction, Multiplication, and Division.

Numeration of numbers to be called so many tens [We

say two tens instead of twenty, three tens instead of thirty, and so on.]——Numeration of numbers to be called so many tens so many units——Notation of numbers up to 99———Numeration and notation of 100——Chanting numbers either in the ascending or descending order——— Multiplication———Division as the inverse operation of multiplication——Division as partition into equal parts—— Review (*3*).

Second school year——Numeration of numbers up to 1000 is to be taught. Mental calculation within the limits of number not exceeding 100, in particular, multiplication of two digits and its inverse operation of division, are to be practised. And thus it is expected that the foundation of the general processes of multiplication and division shall be laid.

I. Addition and Subtraction

Review of the last year's course——Adding a digit (*1*) —— Subtracting a digit (*1*) ——Adding a digit (*2*) ——Subtracting a digit (*2*) ——Adding a digit (*3*) ——Subtracting a digit (*3*) ——Adding a two-figured number (*1*) ——Subtracting a two-figured number (*1*) ——Adding a two-figured number (*3*) ——Subtracting a two-figured number (*2*) —— Adding a two-figured number (*3*) ——Subtracting a two-figured number (*3*) ——Numeration of numbers up to 1000 ——Notation and different ways of reading numbers up to 1000 ——Calling aloud numbers, each differing from the preceding one by 10, either in the ascending or descending order——Calling aloud numbers, each differing from the preceding one by 1, either in the ascending or descending order.

II. Multiplication

Multiplication table for multiplication by 2 ——Ditto by 3 ——Ditto by 4 ——Ditto by 5 ——Ditto by 6 —— Ditto by 7 ——Ditto by 8, 9 ——Multiplication table for

multiplication by 1 ——Multiplying by 10 and 100 —— Multiplying a number, so many tens or so many hundreds, by 2, 3 ——Ditto by 4, 5 ——Ditto by 6, 7 ——Ditto by 8, 9 ——Multiplication, such that, being multiplied by a digit, each partial product comes out not greater than 9.

III. Division

Division by 2, 3 ——Ditto by 4, 5 ——Ditto by 6, 7 ——Ditto by 8, 9 ——Division by 1, 10, 100 —— Division with remainder——Division such, that, being divided by a digit, the quotient comes out to be a number, so many tens or so many hundreds——Division, such that each figure of the dividend is divisible by one and the same digit——Review.

Third school year——The main object of this year's course is to teach children the four operations in written work within the limits of numbers less than 10000, and let them acquire readiness and skill in actual calculation. The first Part is allotted to addition and subtraction, the second to multiplication, and the third to division. In division, the divisor is to be either a digit or a two-figured number, but not greater.

I. Addition and Subtraction

Numeration and notation——Mental calculation (*1*) —— Written addition (*1*) ——Ditto (2) ——Ditto (*3*) ——Applied problems——Written subtraction (*1*) ——Ditto (2) —— Ditto (*3*) ———Ditto (*4*) ———Review (*1*) ———Applied problems (2) ——Ditto (*3*).

II. Multiplication

Mental calculation (2) ——Written multiplication (*1*) —— Ditto (2) ———Ditto (*3*) ———Ditto (*4*) ———Applied problems (*4*) ——Written multiplication (*5*) —— Ditto (*6*) ——Ditto (7) ——Ditto (*8*) ——Applied problems (*5*) —— Review (2).

III. Division

Mental calculation (*3*) ——Written division (*1*) —— Ditto (*2*) ——Ditto (*3*) ——Ditto (*4*) ——Ditto (*5*) —— Mental calculation (*4*) ————Review (*3*) ————Applied problems (*6*) ——Ditto (*7*).

Appendix——Tables of Compound Numbers.

It may not be altogether superfluous to insert at this point a remark concerning a distinctive feature of our numeration. In occidental countries, in naming a number to be expressed by several figures, the row of figures is divided into periods, each consisting of three figures, and every period has a distinct name, such as units, thousands, millions, billions, etc. In our numeration, a period consists of four figures instead of three, and each period has a distint name; thus we have a special name for ten thousand, viz. *man*, and, instead of counting by thousands up to a million, we count by *man* up to *man* times *man*, i. e. hundred millions, to which we give a distinct name of *oku*. We leave out mentioning here the names of the periods after *oku*, as, in actual practice, it will be seldom necessary to name numbers exceeding *okus*.

Fourth school year——The four operations in written work are to be further practised within the domain of numbers less than a *oku* i. e. hundred millions. In particular, division by numbers expressed by three or more figures are to be taught. Further, simple calculation in compound numbers and decimals are to be taught and practised. The first Part is assigned to integers, the second to compound numbers, and the third to decimals.

I. Integers or Whole Numbers

Numeration and notation———Mental calculation——— Addition———Subtraction———Miscellaneous problems (*1*)

——Multiplication——Division (*1*) ———Division (*2*) —— Miscellaneous problems (*2*).

II. Compound Numbers

Measures of length——Measures of distance——Measures of area——Applied problems (*1*) ——Measures of capacity ——Measures of weight——Applied problems (*2*) ——Money and currencies——Measure of time——Applied problems (*3*).

III. Decimals

Numeration and notation——Mental calculation (*2*)—— Addition of decimals———Subtraction of decimals——— Multiplication of decimals———Division of decimals——— Miscellaneous problems (*3*).

Appendix—Tables of Compound Numbers.

Before proceeding further, it will be convenient to make a few remarks here, as they apply to the fifth school year as well as to the sixth. In actual practice, teachers are allowed to introduce such modifications or alterations as they may think fit or desirable in view of the *de facto* state of development of children's aptitude for learning and other circumstances of similar nature. In particular, the distribution of the subject-matter to be taught among the three Parts, into which one year's course is sub-divided corresponding to the three terms of a school year, need not be strictly adhered to. There are certain matters which are subject to variation according to time and locality; in teaching such matters, children are to be made aware of the change, or are to be taught after revising them. In applied problems, numbers were so chosen as to be as near as possible to the actuality of everyday life, and, as a consequence, some of them involve rather complicated calculations; in such cases, it is recommended to cut short the calculations and to be satisfied with approximate results, provided we can do so

without endangering the purpose with which such problems were put in. In schools where soroban-calculation is taught, a part of the text is to be omitted at the discretion and convenience of teachers.

Fifth school year——The object of this year's course is to give children proficiency in calculations involving integers, decimals, and compound numbers, and in the solution of applied problems. As regards compound numbers, those in denary system are allotted to the first Part, those not in denary system to the second, and lastly, the metric system and some of the more important of foreign weights and measures to the third. Besides, some simple and easy mensurations are also to be taught. Answers to problems which are very simple are left out. Answers to other problems are given in an abbreviated form, so that teachers are required to put them into a proper form before making use of them in class work.

I. Integers and Decimals

Numeration and notation——Addition——Subtraction——Applied problems (*1*)——Multiplication (*1*)——Division (*1*)———Multiplication (*2*)———Division (*2*)———Applied problems (*2*)——Lengths——Areas (*1*)——Volumes (*1*)——Capacity——Weights——Money and currencies.

II. Compound Numbers

Distance——Applied problems (*3*)——Surveyor's square measure——Area (*2*)——Applied problems (*4*)——Time——Applied problems (*5*)——Applied problems (*6*).

III. Compound Numbers

Metre—— Areas (*3*)—— Volumes (*2*)——Litre——Gramme——Applied problems (*7*)——Foreign weights and measures——Applied problems (*8*).

Appendix——Tables of Compound Numbers.

Sixth school year——In this school year, simple calculations in fractions and percentage are to be taught, and, in addition, the whole of what has been thus far taught, is to be reviewed. To the first Part is apportioned fractions, to the second percentage, and, lastly, to the third reviews. In the second part, percentage is to be taught, after having made the conception of ratio clear and definite. As to the answers to problems, what has been said concerning the year preceding, applies to this year also; moreover some of the answers are given to such number of decimal places as may not be necessary, and before making use of them for class work, they are to be so rounded as to suit the nature of problems by simply cutting off unnecessary figures, or by *four-off-five-in* rule, or by some such conventional process. *Four-off-five-in* is the name given to the usual practice of rounding numbers by cutting off figures simply, if the first figure of the part to be cut off is not greater than 4, and adding 1 to the last figure retained, if the first figure just spoken of is not less than 5.

I. Fractions

Multiples and measures or divisors——Notions and notation of fractions——Simple calculations in fractions——Different kinds of fractions——Reduction of fractions——Addition of fractions (*1*) ——Subtraction of fractions (*1*) ———Reduction of fractions to a common denominator———Addition of fractions (2) ———Subtraction of fractions (2) ——Applied problems (*1*) ——Multiplication of fractions (*1*) ——Division of fractions (*1*) ———Multiplication of fractions (2) ——— Applied problems (2) ——Applied problems (*3*) ——Division of fractions (2) ———Applied problems (*4*) ———Applied problems (*5*) ——Reduction of a decimal to a common fraction——Reduction of a common fraction to a decimal ——Applied problems (*6*).

II. Rate-calculation (Percentage)

Ratio——Problems involving ratio (*1*) ——Problems involving ratio (2) ——Notion and naming of rate——Relation

between base, *ratage*, and rate. [What is here named *ratage* has been hitherto purposely called "percentage," as it was thought desirable to do so, as long as we are concerned with giving a general idea; but there is a shade of difference in meaning between the two. *Per cent.* or, fully written, *per centum* signifies "by the hundred," and the word *percentage* is apt to give prominence to the number 100. With us, such is not customary. In this respect, we give, comparatively speaking, rather more preference to the number 10 than to 100; in fact, we do not give any decided prominence to any power of 10. Under such circumstances, it was thought undesirable to use the word *percentage*; so I was obliged to coin a new word *ratage* in absence of any proper English equivalent for what we call *buaidaka*. *Base* multiplied by *rate* gives *ratage*, just as *base* multiplied by *rate per cent* gives *percentage*.]——Problems in rate———Profit and loss———Land tax———Income tax———Various taxes and duties——Interest——Bonds and shares——Miscellaneous problems.

III. Review

Integers and decimals———Compound numbers———Problems in mensuration——Fractions——Problems in ratio——Problems in rate.

Appendix——Tables of Compound Numbers.

The Higher Elementary Course

As explained in the first chapter, the higher elementary course which extends over two or three years, does not belong to the main trunk of our *educational tree*. And it is our purpose to go, in the first instance, through the trunk system of our educational organization. However, as regards the State text-books on arithmetic, those for the higher elementary course are so affiliated to those for the elementary course i. e. the ordinary elementary course that, in a certain sense, the former may be looked upon as the continuation of the latter. On this account, we will give immediately below the list of contents of Teacher's Books on arithmetic for the higher elementary course.

Most of the remaks made regarding the fifth and the sixth years of the elementary course, apply to all the years of the higher elementary course.

First year——In this year, calculations in integers, decimals, compound numbers, and fractions, and rate-calculation, are to be reviewed; in particular, calculation in fraction and rate-calculation are to be pushed on to a more advanced stage, and, further, solution of problems in simple proportion are to be taught making formal use of proportion. Here again a year's course is divided into three Parts. To the first Part is allotted integers and compound numbers; to the second fractions and rate-calculation, and lastly, to the third proportion.

I. Integers, Decimals, and Compound Numbers

Numeration and notation——Addition and subtraction ——Multiplication——Division——Applied problems (*1*)—— Metric system of weights and measures——System of weights and measures in common use——Measure of time——Money and currencies——Reduction of a compound number to a

number of single denomination——Reduction of a denominate number to a compound number——Addition and subtraction of compound numbers———Multiplication and division of compound numbers———Foreign weights and measures——Applied problems (*2*).

II Fractions and Rate-calculation

Multiples and measures or divisors——Greatest common divisor——Least common multiple——Reduction of fractions to lower terms———Reduction of fractions to a common denominator———Addition and subtraction of fractions——Multiplication and division of fractions————Applied problems (*3*) ——Rate——National taxes——Bonds and shares ——Simple interest——Compound interest.

III. Proportion

Ratio——Proportion——Problems in direct proportion ———Problems in inverse proportion———Problems in distributive proportion or division into proportional parts ——Applied problems (*4*).

Appendix——Tables of Compound Numbers——Table of Compound Interest.

Second year——In this year, we begin with reviews in simple proportion and proceed to solution of problems in compound proportion and alligation. Besides, the whole of what have been thus far taught, is to be reviewed and supplemented. To the first Part is assigned proportion, and to the second and the third reviews.

I Proportion

Ratio and Proportion——Problems in proportion (*1*) —— Compound ratio and compound proportion——Problems in compound proportion———Chain rule———Problems in proportional parts———Problems in alligation——Problems in proportion (*2*).

II Review

Integers and decimals———Compound numbers——— Measures and multiples——Fractions——Problems in length ——Problems in area——Problems in volume——Problems in capacity——Problems in weight——Problems in money and currencies——Problems in time and calendar——Arc and angle——Longitude, latitude, and standard time—— Temperature——Graphic plotting and representation.

III Review

Problems in rate———Problems in national taxes——— Insurance———Problems in simple interest———Problems in bonds and shares———Discount————Problems in compound interest——Problems in savings and bank deposits ——Promiscuous examples.

Appendix——Tables of Foreign Weights and Measures—— Tables of Foreign Money and Currencies——Table of Compound Interest——Table of Saving Deposits.

Third year——In the first and the second terms are to be taught mensuration, square root, and cube root; and the third term is to be devoted to reviews covering all the grounds thus far gone over. Soroban-calculation and easy book-keeping for everyday use may also be included, in which case a part of the regular text is to be omitted at the discretion of teachers.

I Mensuration

Angle——Parallel lines——Triangle——Quadrilateral—— Polygon——Similar plane figures——Areas of a rectangle and a square——Extraction of square root——Extraction of square root of a decimal and of a fraction——Area of a triangle——Relation between the three sides of a right-angled triangle——Areas of a parallelogram, a rhombus, and a trapezoid————Area of a polygon————Promiscuous examples (*1*).

II Mensuration

Circle——Inscribed and circumscribed figures——circumference and area of a circle——Ellipse——Relation between two planes in space——Relation between a straight line and a plane in space——a right parallelepiped (parallelopiped) or a rectangular parallelepipedon (parallelopipedon)——Cube——Extraction of cube root——Extraction of square root of a decimal and of a fraction——Prism and cylinder——Pyramid and cone——Frustum of a pyramid and of a cone——Sphere——Promiscuous examples (*2*).

III Review

Miscellaneous applied problems in four operations——Problems in proportion——Problems in rate-calculation——Problems in interest———Promiscuous examples (*3*) ——— Arithmetical and geometrical progressions.

Appendix——Table of Compound Interest——Table of Saving Deposits.

There still remains one more volume of State text-books on arithmetic. It is the Teacher's Book on soroban-calculation. In the above syllabus of the higher elementary course, we find here and there the statement that soroban-calculation may be taught; and this volume is to supply the subject-matter to be taught arranged in a proper order, in case teachers avail themselves of the above permission. Now, in view of the opinions among educationalists elucidated before, soroban-calculation ought not to furnish the contents of a separate isolated volume. It should find itself scattered all over and permeating throughout the entire course. In short, arithmetic including soroban-calculation should form an organic whole. Indeed, it is in this direction that we look forward, first of all, for the future progress of the teaching of arithmetic in elementary schools.

A series of *brochures* entitled "Prospectus for the Compilation of State Text-books" was published by the Department of Education. There are two series, which will be here designated as the old series and the revised series. In the old series, arithmetic is treated in a supplementary volume, while, in the revised series, the whole of the third volume is exclusively devoted to arithmetic in the elementary i. e. ordinary elementary course. It may be worth while to notice that the former was published in 1903 very soon after the State text-book system came into existence, and the latter was issued in 1910 some time after the term of compulsory or obligatory education was prolonged by two years.

Formerly, beginning with six years of age of normal children, we had four years of the elementary course which was compulsory, and, above it, another four years of the higher elementary course which may be lengthened by one or two years. But, since April 1908, the first two years of the higher elementary course were shifted into the elementary course and made compulsory. Thus, at present, the elementary course extends over six years and is compulsory, and above it stands the higher elementary course of two years which may be prolonged to three years. The revised Prospectus for the Compilation of State text-books on arithmetic is chiefly concerned with the alterations made necessary so as to accord with the change in the duration of compulsory education from four years to six.

Between the State text-books on arithmetic first published and the revised ones which are now being used, there is not much difference worthy of special attention save in one point which will be forthwith explained and which, as it seems to me, is of fundamental importance. This refers to the manner and grading in which the domain of numbers treated is enlarged up to 1000. The manner and grading in which the domain of numbers treated is extended, will, for brevity's sake, be called "the gradation of extension of number-domain."

According to the old syllabus, the number-domain was extended only to 20 during the first school year; the first term was assigned to numbers up to 10, the second to numbers up to 19, and the third to numbers up to 20. And in the second school year, the number-domain was extended to 100 from the beginning and stopped at that point till the end of the year. Thus the revised syllabus shows more boldness in the extension of number-domain; in the first school year already is this domain extended to 100 and in the second to 1000, as is to be seen from the list of contents of text-books given before. Now there is one thing which might be mentioned in this connection, and that is, that our numeration is as simple and regular as it can possibly be, following exactly the lines indicated by the Arabic notation. In particular, as regards numbers below 100, ours compare very favourably with such as the one, in which our ten-three is called *thirteen*, or our two-ten-five *fünf und zwanzig*, not to say of the still more elaborate numeration as the one, in which our seven-ten-nine is named *soixante-dix-neuf*. This extreme simplicity of our numeration was one of the important considerations, if not the most important, which lead to the aforesaid extension of number-domain in the first two school years. Concerning this alteration, however, there was much difference of opinions even within the Department of Education itself; and, if I am rightly informed, it was decided at hair's breadth after heated discussions in favour of the one actually adopted. Thus by no means is it a settled question; on the contrary it is only just that we should consider the revised syllabus as an experimental one. Before proceeding further, may it be allowed to say a few words in general regarding the gradation of extension of number-domain.

Gradation in the Extension of Number-domain

That arithmetic can be learned if taught, is the common inheritance of children. A few lack this faculty, just as some are colour-blind. Probably a still smaller number show singular acuteness of arithmetical instinct and produce marvellous results, themselves scarcely knowing how. But both extremes are rare exceptions, and intermediate between them, there are millions who can learn arithmetic. Now these millions may, broadly speaking, be classified into two groups; those belonging to one group do not dislike arithmetic, though they may not be fond of it, while those of the other group somehow can not reconcile themselves to lessons in arithmetic, although they learn it under compulsion. A large number, perhaps the majority, of those belonging to the latter group, are often said to possess inclination for literary subjects, such as languages, history and so forth——subjects which are not seldom designated as non-mathematical, although there is no real reason why they should be put in contradistinction to mathematics. Now in all cases where innumerable individuals divide themselves into two groups, there are those who are born into a particular group as well as those who might have gone to either the one or the other of the two groups. Very often some merely casual circumstances, and still more often some trivial conditions at the start, decide the destiny of an individual as to which of the two groups he will belong.

Needless to say, a good start is always essential for success, and particularly so in the case of the teaching of arithmetic which begins at the very beginning of education properly so called, and covers a period lasting many years during which the children's mental development as well as bodily growth are at the height of vigour and activity. These considerations show the difficulty of overestimating the importance of the ways and means, by which arithmetic

should be taught at the very first stage of elementary education. And nothing seems to play a more important *rôle* in teaching the beginning of arithmetic than gradation in the extension of number-domain up to 100 or 1000.

Some ten years ago, I was somehow led to study a little the question of how to enlarge the domain of numbers up to 100 starting from the very beginning of primitive counting. Speaking in general, syllabuses in four rules may vary multifariously between two extremes. On the one hand, following the order of addition, subtraction, multiplication, and division, each fundamental process may be exhaustively taught irrespective of number-domain before going on to the next. This may provisionally be called the *unmixed* method. On the other hand, we may give an all-round treatment (*Allseitige Zahlenbehandlung*) to each number in succession, by which is meant, that the four fundamental processes are taught with each number before the next number is taken up. This, I believe, is often called the Grube method. Now, limiting ourselves to the number-domain up to 100, we have here again a variety of syllabuses fluctuating in manifold ways between the two extremes just mentioned. Without making any special effort, I was able to catalogue more than twenty such syllabuses. None of them is merely hypothetical; they were all taken from booklets bearing some such names as *Junior Arithmetic*, *Éléments d'arithmétique*, and *Rechenbuch*, to be actually found in a corner of my own library. A few of them were found to be very much alike; and after all, there remained fifteen distinct syllabuses, of which a comparative study was made and the result put down in a memorandum. This memorandum was not originally intended for publication; but some time afterwards, on the occasion of the twenty-fifth anniversary of a private school called "Tokio Butsuri-Gakko", where mathematics and physics are taught, I was asked at too short a notice to contribute to the commemoration number of

the monthly journal issued by the Alumni Association of the school, an article of general interest on any subject relating to elementary mathematics. I had been all the time admiring the work done by the school for the cause of education, especially in supplying teachers of mathematics for secondary schools, and could not have possibly refused this request on so signal an occasion, although the time at my disposal was altogether too short for writing an article expressly for the purpose. So, I came out of a corner by sending, though reluctantly, the old manuscript just spoken of. It is to be found on pp. 514—548 of the Journal of the Tokio Butsuri-Gakko, Vol. CLXXX, Tokio, 1906.

Perchance I have already gone too far astray from the purpose of this summary report by giving an account of how the above article came to be published, and I would have dared too much, were I to go into detail. Only, may it be allowed to add a few more words. The comparative study of very many different syllabuses led me to conclude that we have to give up the search for any particular syllabus which is the *best* and rest satisfied with having found a few which are *better* than the remaining ones. The simplicity of our numeration is surely an important factor in deciding the question at issue, and there is truth in the argument that we can go, so to speak, much faster in enlarging the number-domain, at any rate, up to 100, if not to 1000, than in countries where numeration is not so simple as ours. And yet, extending the number-domain up to 100 already in the first school year i.e. between the sixth and seventh year of normal children, as is done in the revised syllabus now in use, seems to be not altogether free of weighty objections. My meagre knowledge in such matters and utter want of experience in actual teaching forbid my expressing any decided opinion on this point. Only I venture to suggest that, so long as the inflexible State text-book system is in force, it would be advisable to

have two or three alternative syllabuses, at any rate, for the first two school years. Examples of this are to be found elsewhere, and are, as it seems, working exceedingly well. Moreover, this suggestion has in its favour the consensus of opinion among experienced elementary school teachers, to whom I have submitted my proposal, and who are bold enough as to avow without reserve their conscientious and unbiassed judgment in spite of the high pressure exerted by the State text-book system.

Multiplication Table

As was said before, one of the essential characteristics of our multiplication table is the entire absence of any word signifying "times", so that the two factors and their product are named in unbroken succession. Another no less noteworthy feature is that we do not distinguish the multiplier and the multiplicand, but consider them simply as factors, and that, if they are different, we always call the smaller number first. Thus whether we are multiplying 3 by 4 or 4 by 3, we say invariably "three-four-twelve." The two peculiarities just mentioned are more organically interwoven than would appear at first sight. The absence of the connective renders easy the interchange of the two factors, which, in turn, facilitates putting the smaller of the two numbers first.

Our multiplication table may also be interpreted as giving the result of multiplying together two digits without paying any attention to the particular order in which they are arranged i.e. the product of multiplying either the one or the other of the two factors by the remaining one. For instance, "three-four-twelve" may be looked upon as covering both multiplying 3 by 4 and multiplying 4 by 3. In written work as well as on the soroban, as soon as the eye catches 3 and 4, no matter whether 4 lies above or below 3, or whether 4 lies to the left or to the right of 3, if they

are to be multiplied, at once we think of "three-four-twelve" without the slightest effort. Perhaps, after all, this latter interpretation is more in accord with the real nature of our multiplication table than the preceding one which assumes the omission, for the sake of brevity, of the word meaning "times." The former seems to be in favour among those who are used to skim the surface of such questions but not accustomed to go into them any deeper.

The multiplication table now in use had its origin in the olden days when calculation was done solely on the soroban. On the introduction of written arithmetic, its use was continued. Indeed, it suited the written work as well as the soroban-calculation so admirably that change or modification was not only necessary but found to be absolutely inadmissible. Now we have been speaking of the superfluity of text-books. Some one intending to write a new text-book of arithmetic, might have wished to have an excuse for doing so. It is only natural that he should have searched for some novel features of his book, of which he might boast. Owing to some such trivial circumstances, the modification of the multiplication table which consists in including for instance "four-three-twelve" side by side along with "three-four-twelve", was proposed from time to time, but was fated to fall flat as often as and as soon as it was recommended. Arguments which were artfully devised in favour of the said modification, may possibly apply to the case of professional experts engaged in astronomical or other complicated calculations, but not to calculations of everyday need for all people. In discussing matters relating to general education in the sense that it appeals to the so-called larger public, an argument which applies only to an exceedingly small part——a mere drop in the ocean of water——but not to the whole, is misleading and sometimes does more harm than a positive misstatement would do.

The extreme simplicity of our multiplication table, mainly

due to the two characteristic peculiarities just elucidated, seems to be worthy of recommendation to other nations whose multiplication table is far more elaborate than ours. However, the caution must be observed that one must try his best to throw off the restraint of long habit in order to be able to pass an unbiassed judgment upon such questions of the commonest concern. It may be difficult to learn, but it is infinitely more difficult to unlearn. We can not help admiring the truth underlying the saying:

> For only by unlearning Wisdom comes,
> And climbing backward to diviner Youth.

Decimals introduced early

As regards the position of *decimals* in the range of subject-matter of instruction in arithmetic, there have been many discussions and controversies. In former days when we blindly followed some of the English and American text-books of arithmetic or their translations, it was customary to teach fractions first and then decimals considered as special types of fractions. Again there were, and there are still to this day, a few rigourists who say "decimals are fractions"——there is no doubt about that——but then, suddenly jumping over a wide chasm, insist upon that therefore decimals should be treated as particular cases of fractions. They are the men who can not possibly reconcile themselves to any thing but what can be explained or proved. Now very often it happens, that the simpler the matter is, so much the more difficult it is to explain. A rigourist may lose himself in joy for having found an exquisite explanation of something, which is only clear to himself but to nobody else. Once a philosopher questioned a mathematician: How do you explain *one*? The latter answered simply and without a moment's hesitation: *One* is *one*. The querist pushed on persistently: You explain *two* by saying *two* is one plus one; now how do you

explain *one*? The mathematician was obstinate as ever and answered once for all: *One* is *one* because *one* is *one*. Let the rigourist catch a glimpse of the latent truth lurking in the above dialogue, and to use an eloquent vernacular expression, his thought would have already penetrated half way through.

In elementary education, particularly in its first stage, sometimes knowledge is to be implanted in the juvenile mind in such a way that the child knows without thinking why it must be so. And in due course of time, he will come, if properly led, to see why it must be so of many things which he already knows. In thus properly and suitably leading the acquisition of the child's knowledge, which should keep harmony with the development of his intelligence, lies the secret of the art of teaching. If any name be necessary for such a method of teaching, it may perhaps be called *ultra-heuristic*. It may not be superfluous to add that sometimes a child comes to see the reason for a certain statement so gradually and in such a round-about way that he scarcely knows how, and sometimes it flashes upon him of a sudden and yet not abruptly.

The above were some of the important considerations which led to the final adoption of the syllabus, in which decimals are introduced as soon as possible as an extension of the denary system of notation, and examples of which are to be found in several of the European continental countries. In fact, as is to be seen from the contents of the State text-books of arithmetic, decimals are taken up very early, and fractions are put off till very late and not much importance is attached to them. Our experience has shown that children have no great difficulty in comprehending decimals, and fortunately for us most of our concrete numbers are in denary system, so that there is not much need to teach fractions for everyday calculations

Concerning the teaching of arithmetic in elementary schools, there still remain several points, which I would have gladly noticed were it not for lack of time at my disposal. I should like also to say something about simple mensuration and easy book-keeping. But I have already given more space to the elementary education than I could well spare; so I leave them all for some future opportunity and hurry on to the next chapter which will treat the teaching of mathematics in Middle Schools.

The following is to be added to the list of the works of reference given at the end of the first chapter.

Annual Report of the Minister of State for Education. Published by the Department of Education.

This report which is an abridged reproduction in English of the Japanese original, is published once every year, the latest being the thirty-sixth published in July 1911 and containing the record of educational affairs for the statistical year 1908-9.

The Full Recognition of Japan. By Robert P. Porter. Published by Henry Frowde, Oxford University Press, London, 1911.

This work which came to my notice while the present report was in the course of printing, contains two chapters IX and X [pp. 153—194] devoted to education, in which slight inaccuracies are to be observed.

CHAPTER IV

THE TEACHING OF MATHEMATICS IN MIDDLE SCHOOLS

A boy who has finished the elementary school course of six years or has equivalent qualifications, may enter a middle school. The middle school course extends over five years. Thus, in normal cases, a boy enters a middle school when he is just twelve years of age and leaves it just before he reaches his seventeenth year. A supplementary course of not more than one year may be added, to which graduates of middle schools may be admitted.

The whole country excluding Taiwan (Formosa), Chōsen (Corea), and Karafuto (Japanese Saghalien), which is sometimes called "Japan Proper" and sometimes "Japan Old," has an area of a trifle less than 150,000 square English miles, i.e., as may be noticed in passing, about as many square miles as there are elementary school teachers. (See p. 3 bottom.) So far as the population is concerned, the growth seems to be fairly regular, the average rate of increase being about 1.35 per cent. per annum. The estimated population of Japan Old in 1912, is about 26½ million males and 26 million females, and the total 52½ million. The average number of persons per family is something between 5 and 6.

Now, Japan Old is administratively divided into 47 prefectures, of which the largest but the most thinly populated (Hokkaidō) is called "Dō", the three containing the larger cities of Tokio, Osaka, and Kioto respectively, "Fu", and the remaining forty three "Ken." Each prefecture is divided into a number of municipalities (*shi*) and counties (*gun*), the distinction between the two consisting in that a

municipality either contains, or is itself, a city. Each county is further divided into a number of towns (*chō*) and villages (*son*). The county which is a remnant of the antiquated division of the whole country into so many provinces (*kuni*) and each province into so many counties, occupies rather an awkward position on the lineage of the local administrative system, and there has been even talk of its suppression. So in many instances, it is at least convenient to consider each prefecture to be administratively divided into a number of municipalities, towns, and villages.

Each prefecture is placed under obligation to establish and maintain one or more middle schools, the Minister of Education having the power to dictate the increase of the number of middle schools when he deems it necessary. In actual practice, there has been found no need for the exercise of such an authority. Middle schools may also be established by counties, municipalities, towns, villages or a union of them formed for the purpose, provided there is no fear of influence prejudicial to the carrying out of the elementary education scheme within the district in view of financial resources. Middle schools may also be established by private individuals. In every case, a middle school cannot be established or discontinued without the sanction of the Minister of Education. Schools which are not established in accordance with the regulations of the Middle School Ordinance issued in 1899 and slightly revised in 1907, cannot be called *middle schools*.

A branch school may be established for the convenience of boys who live at too great a distance from the main school to go to it daily, and yet are too young or otherwise find it inconvenient to leave their homes. Such a branch school is not to have any class higher than the third year. Formerly it was allowed to attach more than one branch school to one main school; but plausible as the scheme may appear, somehow, it did not work well and gave rise

to much abuse. So one branch only to one main school is now allowed, and even this is not encouraged. Consequently the number of branch schools has been constantly on the decrease.

According to the official return made in April 1912, there were 314 middle schools, of which 307 were main and the remaining 7 branch schools. Of these, 2 were government institutions; 238 main and 6 branch schools were public institutions; and 67 main schools and 1 branch school were private institutions. The number of classes was somewhat over 3 thousand, the number of teachers a trifle less than 6 thousand, the number of pupils about 120 thousand, and the number of those who graduate at the end of a school year a little over 16 thousand. It may be worthy of notice that we have now nearly 50 more middle schools than as was anticipated in what was at that time considered as an utopian scheme of educational organization embodied in the first Educational Code of 1872. It may also be added that we have now about one middle school to every 80 thousand of the male population.

The so-called "Revision of the Educational System", whose avowed aim is the shortening of the number of years required in going through the trunk system of our *educational tree*, is a question of such magnitude and complexity that it has taxed the best efforts of our educationists both within and outside the Department of Education for a considerable time and as yet has found no feasible solution. For several generations of the Ministers of Education, every one of them formulated some sort of revisionary scheme, but all these schemes were destined to be shelved before they had any chance of being actually adopted. The very first dream of a new Minister of Education may be said to have been finding himself hailed as the great man who has accomplished this difficult task which defied the power and ability of his predecessors. For the cause of education, it is very un-

fortunate that this work, upon which the nation's future welfare so much depends, should have taken such a course as to become the object of bureaucratic, if not personal, ambition.

Toward the close of July 1911, the revision of very many regulations which have either direct or indirect bearing upon carrying out the Middle School Ordinance and of the syllabuses of instruction in middle schools, received its final authorization. It is the outcome of polemics which dragged on for more than a year. Now, unanimity of views in such a matter is altogether out of question; but, at any rate, any real improvement must be based upon the dominating opinions of those who really understand educational intricacies. Unfortunately, such was not the case in the present instance. Indeed there has been a rumor which may be interpreted in the sense that even bureaucratic pressure was brought to bear upon antagonistic views strongly backed by those whose voice in educational affairs commands the highest respect. It is true that the scheme has undergone modifications while it was maturing; but these alterations were due more to the sheer necessity of circumstances and headlong persistency than to intelligent considerations and profound convictions. [Cf. "Fifty Years of New Japan", Vol. II, Chap. IX, Note, pp. 173-4.]

Were it permissible to dogmatize on a subject so replete with uncertainties as this revision of the educational system, it might be said that its underlying feature is anxiety concerning its workability——an anxiety born of incipient doubt. Those parts of the revised regulations which have direct bearing upon the middle school education are to take effect in April 1912; but as to the entirety of the revisionary measures there has been a talk of their postponement and even of repeal. So it was thought desirable that in this report we should confine ourselves to the actual conditions prior to the date just mentioned.

It may be observed that, using the word "secondary" in the average sense as is used in most of the principal countries of the world, the middle school education does not cover the whole of our secondary education. It may also be added that, so far as the subjects taught and the syllabuses of instruction are concerned, we have but one sort of middle schools. This is in contrast with the conditions in other countries, where there are very often several kinds of schools corresponding to our middle schools. There has been talk among educationists of having two or three sorts of middle schools after the model of German system; but it will take some time before the proposal takes any definite form, not to speak of its adoption.

Aim of, and Subjects Taught and Hour Table in, Middle Schools

The object of middle schools is stated to be "to give a higher general education," by which is meant a general education or liberal culture necessary for those who are to be of middle or higher social standing. This definition, even thus interpreted, is necessarily vague, and the actual aim of the schools may better be inferred from the subjects taught in them.

Subjects taught in middle schools are Morals, the Language (Japanese) and Classic Chinese, Foreign Language (Some one of the three, English, German, and French), History, Geography, Mathematics, Natural History, Physics and Chemistry, Law and Economics, Drawing, Singing, and Gymnastics. The number of instruction hours (recitation hours) per week allotted to each subject in successive years is as follows:—

SUBJECTS	1ST YEAR	2ND YEAR	3RD YEAR	4TH YEAR	5TH YEAR
Morals	1	1	1	1	1
The Language and Classic Chinese	7	7	7	6	6
Foreign Language	6	6	7	7	7
History and Geography	3	3	3	3	3
Mathematics	4	4	4	4	4
Natural History	2	2	2	1ST AND 2ND TERMS 2 3RD TERM 1	*
Physics and Chemistry	*	*	*	1ST AND 2ND TERMS 3 3RD TERM 4	4
Law and Economics	*	*	*	*	2
Drawing	1	1	1	1	*
Singing	1	1	1	*	*
Gymnastics	3	3	3	3	3
Total	28	28	29	30	30

Of the subjects included in the above schedule, Law and Economics may be omitted for the time being and its hours distributed among Foreign Language, and History and Geography. Singing may also be omitted and its hours given to Drawing. One hour more of Drawing may be added in the fifth year at the pupils' request. Hours not exceeding three may be added to Gymnastics [The revised regulation introduces, besides the subjects mentioned above, Industrial Work, which may mean either Agriculture, or Commerce, or Technology.]

The subjects of the supplementary course are to be chosen from among the subjects given above, and each subject may be made optional.

General Remarks Concerning Instruction in Middle Schools

The syllabus of teaching in middle schools issued in 1902, begins with following general directions :—

(*1*) In the middle school, instruction and moral training must go together hand in hand, and thus the object of higher general education is to be attained.

(*2*) The object of teaching each subject must not be lost sight of, and, at the same time, due attention is to be paid to the connections existing among different subjects so as to ensure the harmonious working of the whole teaching.

(*3*) Confusing details must be avoided and empty forms disregarded, so that the pupils may get a correct and clear understanding of the matters taught, and be able to apply freely what they have learned.

(*4*) Text-books should be used whenever possible. Care must, however, be taken to make intelligent use of them and not to be hampered by too closely adhering to them. Care must also be taken in selecting text-books ; and once adopted, they must not be altered unless there be some substantial reasons for doing so.

(*5*) The progress of teaching is to be premeditated and made uniform, so as not to fall into the necessity, due to having been too minute at the start, of hurrying on towards the close of a school year.

(*6*) Although it is impossible to fix the number of working days, they should not be less than seventy-five days in each of the first two terms, and not less than fifty-five days in the third term.

(*7*) Instruments, models, specimens, drawings, etc. used, need not be of fine finish or costly beyond the requirements of teaching. Teachers should make efforts to turn to account articles of daily use, or of their own make,

as much as possible. Those that are common to several subjects should be used in common as far as possible, and need not be provided for each subject separately.

(*8*) Where there are libraries, museums, factories, experimental stations, and the like, they are to be made use of as much as possible.

[In the revised syllabus of teaching which is to take effect after April 1912, the corresponding general directions consist of five articles. Namely, (*1*), (*4*), (*6*), and (*8*) are omitted, and one new article is added. The article just mentioned is new only in form, its gist being found in the preamble which accompanied the previous regulations. It says that the subject-matter and the order given in the syllabus may be modified or altered at the discretion of teachers and need not be strictly adhered to.]

Syllabus of Teaching of Mathematics in Middle Schools

The branches of mathematics taught in middle schools, are arithmetic, algebra, geometry, and trigonometry. Below are given the details of the teaching of different branches:

Arithmetic

First year ..... 4 hours per week

Introductory:

Numeration; notation; decimals.

Integers and Decimals:

Addition; subtraction; multiplication; division.

Concrete Numbers:

Time; metric system; common weights and measures known by the name of *shaku-kwan* system; money and currencies; foreign weights and measures, and coinage.

N.B.——English and other non-decimal systems may be given with or after fractions.

Properties of Integers:

Divisibility; prime numbers; measures (factors); greatest common measure; least common multiple.

Fractions:

Chief properties of fractions; reduction to the lowest terms; reduction to the common denominator; reducing decimals into fractions, and fractions into decimals; addition, subtraction, multiplication and division of fractions.

Ratio and Proportion:

Ratio; proportion.

Second year ..... 2 hours per week

Ratio and Proportion (continued):

Chain rule; proportional parts; alligation.

Ratage (percentage):

Preliminary explanations; ratage; interest; calculations of daily use relating to ratage.

Powers and Roots:

Square and square root; cube and cube root; mensuration.

Algebra

Second year ..... 2 hours per week

Introductory:

Definition of signs; algebraic expressions; associative law; extention of definitions, negative number.

Integral Expressions:

Addition; subtration; multiplication; division.

Equations:

Linear equations with one unknown quantity.

Third year ..... 2 hours per week

Equations (continued):

Simultaneous linear equations with two or more unknown quantities.

Integral Expressions (continued):

Distributive law; factors; greatest common factor; least common multiple.

Fractional Expressions:

Fundamental properties of fractions; reduction to the lowest terms; reduction to the common denominator; addition, subtraction, multiplication, and division of fractions.

Equations (continued):

Equations with one unknown quantity reducible to linear equations; quadratic equations with one unknown quantity; equations with one unknown quantity reducible to quadratic equations; simultaneous equations containing quadratic equations; interpretation and discussion of solutions.

Fourth year ..... 2 hours per week

Irrational Expressions:

Extension of the definition of exponent; irrational numbers; approximate values of irrational numbers.

Ratio and Proportion:

Case of abstract numbers; case of quantities, commensurable and incommensurable; surds.

Progressions:

Arithmetical progression; geometrical progression.

Permutation and Combination.

Binomial Theorem:

Positive integral exponent.

Logarithms:

Fundamental properties of logarithms; logarithmic tables.

Geometry

Third year ..... 2 hours per week

Introductory.

Straight Lines:

Angles; parallels; triangles; parallelograms.

Circle :

Fundamental properties ; angles at the centre ; chords ; angles in a segment ; tangent ; contact of two circles ; loci.

Fourth year ..... 2 hours per week

Circle (continued) :

Inscribed and circumscribed figures.

Areas :

Equality of areas of rectilineal figures.

Proportion :

Definition of equality of ratios ; general theorems deducible from the definition.

Applications of Proportion :

Proportional lines ; similar figures.

Fifth year ..... 2 hours per week

Applications of Proportion (continued) :

Areas ; loci.

Planes :

Straight lines and planes ; solid angles.

Polyhedra :

Prism ; pyramid ; regular polyhedra.

Solids with Curved Surface :

Sphere ; cylinder ; cone.

Trigonometry

Fifth year ..... 2 hours per week

Measurement of Angles :

Sexagesimal system.

Circular Functions :

Circular functions of acute angles ; mutual relations among circular functions ; circular functions of complementary angles ; circular functions of some special angles ; tables of circular functions.

Solution of Right-angled Triangles.

Circular Functions (continued):

General definitions of circular functions; mutual relations among circular functions; changes in the signs and magnitudes of circular functions; circular functions of negative angles; circular functions of supplementary angles.

Formulæ for the Sum of Angles:

Circular functions of the sum and difference of two angles; formulæ for the products of circular functions; formulæ for the sum and difference of circular functions.

Relations between the Sides and the Circular Functions of the Angles of a Triangle.

Use of Logarithmic Tables.

Solution of Triangles.

Measurements of Heights and Distances, and Practical Exercises.

The following points are to be particularly attended to:—

(*1*) In enunciating and proving rules and propositions, use always correct and precise wording, and make boys comprehend them clearly and thoroughly.

(*2*) In arithmetic, it is not sufficient to teach rules simply; boys must be made to practise and to acquire proficiency in correct and rapid calculation.

(*3*) In calculation, the verification of the results is to be encouraged as far as possible, so as to make boys gain self-confidence.

(*4*) Examples chosen for arithmetical exercises should be such as have close relation to ordinary affairs of life and business; in examples of ratage and other like calculations, stress is to be laid on explaining the matter itself with which the calculation has to do.

(*5*) In arithmetic, where it would be difficult to make boys understand fully the basis of a rule, just indicate its

general outline and proceed at once to the rule itself; the full explanation is to be given afterwards in algebra.

(*6*) In algebra, linear equations should not be taught all collected together in one place; easier ones should be given as early as possible, in order to stimulate boys' interest in algebra.

(*7*) In teaching geometry, stress should be laid on the strictness of reasoning. For example, in teaching proportion, the difficulty must not be passed over with incomplete explanation, or slurred over with inexact argumentation; if boys are not sufficiently advanced as to be able to follow rigorous demonstration, it is better to *assume* the whole for a while, and return to the subject later on.

(*8*) Construction problems should be given at places suitable in view of their connection with theorems to be applied.

(*9*) It is preferable to give different forms of enunciation of theorems and their mutual relations, after boys are a little advanced, rather than at the start.

(*10*) In trigonometry, measurements of heights and distances together with practical exercises, should be given as early as possible so as to awake boys' interest in the subject.

(*11*) It is better to make use of the true values of the circular functions first, and afterwards to use logarithmic values.

(*12*) Instruments, models, drawings, etc. to be used in the teaching of mathematics are about as follows:—

> Sun-dial; clock; magnetic compass; metric scales; comparative scales showing the three systems, the metric, the common, and English; balances and weights; metric weights; balance used for weighing heavy things; measures for liquids; measures for dry goods; metric system measures; foreign measures; compasses for drawing circles on the blackboard;

straight edge for drawing straight lines on the blackboard; models of vernier, theodolite, tape-measure, Gunter's chain, flag-staffs, etc. Pictures of our own and foreign coins; models of geometrical solids.

The subject-matter of mathematical instruction cannot possibly differ very much, but the mode and order of treatment and relative importance given to several parts may vary a good deal in different countries. This is especially the case in the teaching of arithmetic, in which, it is but natural that there is more room for difference than in others, as it is so closely associated with the conditions and circumstances of daily life, which differ so widely in different countries.

Now, as was stated in the opening chapter, in the early days of New Japan, the line of cleavage between those who direct and those who follow was obscured by the joyful ardour with which all alike devoted themselves to the acquisition of European, not to say American, things and ideas. Everything foreign was then hailed as perfect. Everything not of our own was indiscriminately introduced without hesitation, not a thought having been thrown upon its adaptability. The elementary mathematics taught in schools which correspond to the middle school of the present day, shared, of course, the same fate. This was particularly unfortunate, though perhaps unavoidable, in the case of arithmetic. All sorts of arithmetic, Dutch, English, American, German, French, etc. were brought in promiscuously; and the worst feature of the situation was, they were all covered by one name *arithmetic* without any qualification.

In those days, instruction in schools corresponding to the middle school of nowadays, was given mainly in foreign languages. In schools where English was used, the English or American arithmetic was taught; where German was used, the German arithmetic, and where French was

used, the French arithmetic. This led afterwards to awful confusion and even to bitter controversies which after all had no real ground. As time went on, however, we came to see that there are different types of arithmetic, each of which is fit for a particular country and that we must sooner or later have the arithmetic of our own.

It would be difficult to overestimate the service done in organizing our education system by two Americans, late Dr. David Murray of New Brunswick, at one time the Adviser in the Department of Education, and Mr. M. M. Scott, now of Honolulu, who have done so much in laying the foundation of our normal school education. It happend that Dr. Murray was a mathematician by profession and Mr. Scott, a graduate of an American normal school, was particularly well versed in mathematics. Partly due to the influence of these two men, but, in the main, due to that English has been from the beginning the predominating foreign language prevalent in our country, our first attempt was based upon American models of those days, but we have been changing since then, gradually adapting the treatment to our needs and so finding the arithmetic of our own.

Before closing this chapter, may it be allowed to add a few words concerning the revised syllabus of instruction. It is to be looked upon as an off-spring or a by-product of the recent "Revision of the Educational System," which is justly regarded with misgivings by those who have the cause of our education at heart. The revised syllabus is to take effect after April 1912 under the ironical circumstances that its main-spring, the "Revision of the Educational System" just spoken of, is rocking on shaky ground. Fortunately, we have in front men who, untrammelled by any commitments in the past, are free to act and will act in pursuance of the best interest of our education. However complicated and even hopeless the present situation may appear, we hope——and we believe there are good reasons

for hoping——that, ere long, we shall be able somehow to emerge out of the difficulty.

The main features which distinguish the revised syllabus from the one hitherto in effect, are one hour more added to the number of instruction hours per week throughout all the five years, about twenty days more added to the number of working days in a school year at the expense of shortening by so many days of the summer vacation, and lastly, but not the least important of all, the congested and not seldom bewildering inter-mingling which pervades some of its parts. All this is not to be wondered at, as it is the out-come of the dubious attempt for shortening the number of years required in going through the trunk system of our *educational tree*, without diminishing the quantity of the material of instruction, other conditions remaining unaltered.

As regards the revised syllabus of mathematics, its characteristic trait is the fusion or mixing up of arithmetic, algebra, and geometry. Now there are people who seem not to understand why arithmetic and algebra exist as separate branches of *school mathematics*. It goes without saying that these two branches are very closely allied; indeed the limit between the two is very often of purely conventional nature. Now, because of this conventional nature, it is possible to differ, and, in fact, it must and does actually differ so much in different countries. To the same cause is due, that it was such a hard task for our pioneer workers in the field to define the limit in such a way as best suits our school conditions. How laborious and painstaking this task was, will be told in the next chapter. The final result is to be found embodied in the syllabus hitherto in use. That this result which has taxed the best efforts of those who examined the question from all points of view, and was consolidated by the intelligent experience lasting nearly two decades, should have been found tumbled down as if by an earthquake on awakening from a revisionary dream, is

indeed painful and even heart-rending for those who have the cause of the middle school education truly at heart. It is more like a revolution than a revision. Human weakness very often manifests itself in such a way as to feel bound to do something, although there may be really nothing to be done or even better to leave them alone as it stands and do nothing. Thus in educational matters, more and more do we feel the need of men accustomed to search for permanent tendencies beneath the happening of the hour.

The syllabus of mathematical instruction hitherto in effect has been working reasonably well. From no less than fifty-four middle schools scattered all over the country, our Sub-Commission received, in response to our request, reports giving the views of the teachers of mathematics. Now, on such a question, it is only natural that there should be diversities of opinion. On the whole, however, they seem to indicate the satisfactory working of the syllabus. As a matter of course, in all such cases, there is always room for improvement, but not in the direction along which the dubious revision in question has taken its course. Nevertheless it is not to be disguised that there is none who are in favour of the revised syllabus. In particular, among the men belonging to the so-called "pedagogic school" are to be found some of the most zealous advocates of the spirit which pervades the revised syllabus. They make an appearance, as if they have more followers than they really have, as they are accustomed to talk in pedagogical language and do talk more than others do.

That connections should be established between, and correlations sought among, different subjects taught in middle schools, is the general desideratum. To a certain extent, it is also applicable to different branches of mathematics taught in such schools. But even such an excellent principle has a limit for its application, and if pushed on too far, would lead to unfavourable and even dangerous results. A mis-

conceived pedagogy might lead to the continued uncertainty on the part of the pupil, the worst feature of which is that it is liable to pass on unnoticed on the part of the prepossessed teacher. A misconceived pedagogy or a pedagogy misapplied is prone to making the teaching merely a form void of substantial contents and cannot possibly fail to lead to what Bergson so happily names "the parasitic education."

Once, the father of a middle school boy, on being informed of the confusing nature of the revised syllabus of mathematics, exclaimed: "Arithmetic, algebra, and geometry are difficult subjects, even when taught separately. Why are they to be made purposely more difficult by mixing them together? Woe to boys of the future!" Now, in this unsophisticated outburst uttered spontaneously upon the spur of the moment, there is to be found more truth than the first impulse would suggest.

The juvenile mind seeks simplicity and simplicity can only be attained by non-mixing or singleness. From this point of view, the various subjects taught in the middle school should be kept separate and isolated as far as the nature of the subjects allows. And this is to be effected even at the expense of repetition, which may appear to a casual onlooker as redundant tautology. For instance, it is highly desirable that some parts of arithmetic should be taught again in algebra. Very often overlapping is necessary for the purpose of separation and attendant isolation. Repetition in such cases is not a waste, as superficial observers are apt to think on a momentary impulse. It is more than amply repaid by the simplicity which is thus achieved.

A subject which can not be analysed into distinct fragmentary parts as the different branches of mathematics ——the subject which consists of inseparable chain of successive steps whose order must be kept strictly intact ——is more difficult for an *average* boy than a teacher in

such a subject who has usually an inclination for it might be able to imagine. The non-comprehension of any one particular step very often obscures the whole, and then there is no way of coming out of the difficulty. Boys who have unfortunately missed a few steps here and there, are hopelessly lost in the whirlpool of troubles existing in not knowing where the difficulties lie.

Intermediate between the correlative tendencies which are much to be longed for under ordinary circumstances and the desiderata for making some certain subjects separate from one another as far as possible, is to be steered the proper course of instruction, which a well-planned syllabus ought to direct. Men of the so-called "pedagogic school" are liable to go to the extreme in giving too much weight to the correlative tendencies. They are liable to forget that even the best medicine, if misapplied, does more harm than poison. They are also apt to condemn repetitions indiscriminately and sometimes go even the length of turning deaf ear to the arguments which advocate repetition in certain cases.

Among our middle school teachers and those who write text-books for middle schools, there are some who do not know how to teach what they know so well, perhaps too well. They are those who, themselves being born talented, had no difficulty in knowing and consequently are unable to imagine the difficulties which an *average* boy has to encounter in learning. Again there are some others who presume to know how to teach what they do not really know or know but superficially. They are those who are exposed to the danger of giving more importance to the method of teaching than to the subject-matter to be taught. The middle school education which has to do with the general culture of the *average* boy, must be freed, in order to be efficient, of the deleterious influence which owes its origin to the views of these diametrically opposite kinds of extremists.

CHAPTER V

THE TEACHING OF MATHEMATICS IN MIDDLE SCHOOLS (*continued*)

The basis of human foresight is the contemplation of the past course of development. It is indeed to be deplored that this was almost entirely lost sight of in the considerations which lead to the revision of the syllabus of mathematical instruction. Let us give a retrospective glance as to how our own arithmetic came to be defined as it is now and as to how the boundary between arithmetic and algebra came to be fixed in a manner suitable to our educational circumstances.

In the years close upon the Restoration, the only schools corresponding to the middle school of the present day, were the so-called "Foreign Language School," the "English Language Schools" and the preparatory department of a certain special school. The Foreign Language School is different from the school bearing the same name of nowadays, and was not an institute where, as its name would suggest, foreign languages alone were taught. It was the school where something like secondary education was given in different foreign languages. There were several distinct courses, in each of which a sort of intermediate education was given after the then prevailing fashion of the country corresponding to the foreign language which was being almost exclusively used. Boys of the lower grade were unable to follow instructions given in a foreign language; so to a foreign teacher was attached a Japanese interpreter who acted as an intermediary between the teacher and the class. Similar remarks apply to the English Language Schools, of which there were several scattered all over the country.

Among German teachers who taught elementary mathematics were Erwin Knipping whose name will always be remembered with gratitude as the originator of our weather prediction organization, Dr. Leopold Schendel*, Westphal, Hilgendorff, and others. The well-known text-books of those days by Lübsen, Hirsch, Cambry, Koppe, Schlömilch, etc. seem to have been in use.

Some of the French teachers who taught elementary mathematics were Poucet, Maillot, Garaud, Pigeon, Lépissier* *, Prospère Fortune Fouque, Rieux, Sarda (of *l'École Centrale*), Dybowski, Stéphane Mangeot, and Gustave Hélix Berson. The three last mentioned were graduates of *l'École Normale Supérieure*. The text-books which were in use, seem to have been such books as the well-known classical *Géométrie* of Adrian Marie Legendre, H. Sonnet's *Premiers Éléments d'Algèbre*, Amiot's *Éléments d'Algèbre*, Charles Briot's *Éléments d'Arithmétique*, J. A. Serret's *Trigonométrie*.

There were quite a number of American and English teachers who taught elementary mathematics, among whom are to be particularly mentioned Horace Wilson, a graduate of an American normal school, Rev. William Edwin Parson, and M.M. Scott of the Honolulu High School who has been for many years the Superintendent of the schools of that city, and of whom we shall have to say a good deal in speaking later of our normal school education.

Among American text-books of arithmetic may be especially mentioned the following three, which were, comparatively speaking, widely used here in those days.

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* In Crelle's Journal of about fourty years ago, are to be found several short articles by L. Schendel. One article in Bd. LXXXII. is dated "Tokio (Yedo) im Dec. 1875", and another article in Bd. LXXXIV. "Tokio (Yedo) im Feb. 1877."

* * Lépissier was an astronomer and seems to have contributed toward encouraging the study of astronomy while he was here in Japan.

Felter's New Practical Arithmetic. [Several editions, 1st ed. 1872] Prepared by Selim H. Peabody, Professor of physics and civil engineering in the Massachusetts Agricultural College. Published by Ivison, Blakeman, Taylor, and Company, New York and Chicago.

Robinson's Progressive Practical Arithmetic. [Several editions, 1st ed. 1858] And Robinson's The Progressive Higher Arithmetic. [Several editions, 1st ed. 1860] Both prepared by Daniel W. Fish and published by the same publisher as the above.

Some years later was introduced "A Practical Arithmetic" by G. A. Wentworth and Thomas Hill, published by Ginn and Company, Boston, which contains traces of the European arithmetic; but this text-book has enjoyed but a very small circulation.

English text-books of arithmetic were very seldom used. Among those which were to be found here and there in those days, may be mentioned the Arithmetic of the then well-known Chambers' Educational Series, "Arithmetic designed for the Use of Schools——to which is added a Chapter on Decimal Coinage" by Right. Rev. J. W. Colenso, Lord Bishop of Natal, revised edition 1875, "A Treatise on Arithmetic in Theory and Practice——with an Appendix containing an Introduction to Mensuration" by James Thomson, Professor of Mathematics in the University of Glasgow, sixty-ninth edition 1876. The two last mentioned were published by Longmans, Green, and Company, London.

May a few words of passing comments be allowed to be inserted here. In England too, the days seem to have been long past when men like George Peacock, Augustas De Morgan, and the father of Lord Kelvin just mentioned, who distinguished himself as a very successful teacher of mathematics, devoted a part of their busy hours to the writing of text-books of arithmetic for the benefit of the younger

generation.

Text-books of elementary algebra most commonly used in those days, were to begin with, Robinson's Algebra, of which there were two kinds designated "New Elementary" and "New University" respectively, and a little later, Isaac Todhunter's "Algebra for Beginners" and "Algebra for the use of Colleges and Schools" by the same author. The first text-book of geometry commonly used, was Robinson's Geometry, Plane and Solid in separate volumes, which was in time superseded by Chauvenet's Geometry, Wright's Geometry, Wilson's Geometry, and Todhunter's "Euclid for Colleges and Schools."

In the later part of the period, of which we have been speaking, there was a certain College of Engineering under the Department of Public Works, then in existence and abolished in 1885. In this College which has since become one of the most important nuclei of the present Faculty of Technology of the Tokio Imperial University, there were several courses of study requiring preparatory instruction in mathematics, such as civil and mechanical engineering, electrical engineering, and the like. Various branches of mathematics ranging from elementary algebra to the elements of calculus were taught by an English teacher called David H. Marshall, who, if I am rightly informed, now lives in Canada. The text-books used were nearly all of Todhunter's series. Professor John Perry was then professor of mechanical and civil engineering in this College, and his name has been here remembered coupled with that of late Professor W. E. Ayrton who was his colleague.

In the institution, whose name has been changed several times, and, which, after absorbing several other institutions of similar kind, finally gave birth to the present Imperial University of Tokio, were also several courses of study requiring preparatory mathematical knowledge. Robert Henry Smith, a Scotchman, was professor of mechanical

engineering; James R. Wasson, an American, was professor of civil engineering; and Winfield Scott Chaplin who afterward became the Chancellor of the Washington University in St. Louis, and is now the President of the American Rio Grande Land and Irrigation Company, was also professor of civil engineering. American text-books of higher mathematics by Albert E. Church, Professor of mathematics in the U.S. Military Academy, and published by A.S. Barnes and Company, New York, Chicago and New Orleans, seem to have been in favourite use. They were Elements of Analytical Geometry, Elements of Descriptive Geometry in two volumes, and Elements of the Differential and Integral Calculus. Professor Chaplin was wont to teach elements of differential and integral calculus using as text-book Church's Calculus. A little later, the same institution had the good fortune of having, among its teaching staff, Professor J. A. Ewing as professor of mechanical engineering and physics, of whom may it be allowed to add that the writer of the present report has always felt a real pleasure in thinking of himself as having been one of his most devoted and grateful old pupils.

The professors above named taught, besides their specialities, higher applied mathematics, but had nothing to do with the teaching of middle school mathematics, save in an indirect way in that a few of their pupils might have exercised some influence upon the moulding of the present system of mathematical education in middle schools as well as in technical schools. However, I thought it only just, that, somewhere in this report which deals with mathematical instruction of all grades and of all sorts, their names should be mentioned and remembered with gratitude even by those who do not know them personally.

I take this opportunity for noticing the French influence upon the mathematics taught in our country exercised

through the medium of our military education, which was, at the start, modelled on the French system. This fact seems hitherto to have escaped befitting notice. French officers were engaged to teach at our military schools. Quite a number of our officers were sent to France for continuing their study of military sciences. The dominating foreign language used in military schools was of course French; and mathematics which was necessary for artillery officers, sappers, and engineers, was mainly derived from French sources. Even the mathematics taught nowadays in military schools, has something of French flavour. In the early days, among a large number of men who passed through military service, there were found many who knew something of mathematics which may ultimately be traced to the French source. In this way, some meagre knowledge of French elementary mathematics was disseminated, without being particularly noticed, through wider strata than the military circle.

Outside the schools where foreign languages were used as the vehicle of instruction, arithmetic was taught by means of books written in Japanese, which contained nothing but exercises and problems without any explanation or illustrative examples. Problems were derived from various sources; some were obtained from calculations of daily need; some from the mathematics of the old Japanese school; some from the Dutch arithmetic which had already been introduced, though restricted to the narrow circle of the so-called mathematicians of the occidental school of the day, prior to the Restoration; and the rest from all other available sources. Compilers of such books seem to have taken pride in the large number of problems collected indiscriminately, slight regards having been paid to their quality or merit. Such a title as "Three-thousand-Problems" was very common, and, somewhat later, they thought to have improved upon

their former efforts by increasing this number to ten thousand.

That the secondary education of our boys had to be given in a language other than their mother tongue, was an abnormal state of affairs due to the sheer necessity of unusual circumstances, and was to be done away with at the earliest opportunity. Thus instruction given in foreign languages was being gradually transformed into one in our own language. Meanwhile a few text-books of arithmetic appeared in Japanese. However, they were not the outcome of intelligent compilation, but were the result of translating almost literally some of the foreign text-books of arithmetic mentioned above. Some one translated a foreign text-book of arithmetic, not because he knew any thing of mathematics, but because he happened to know the language in which it was written. The Japanese text-books of arithmetic of those days have done perhaps more harm than good, in as much as they gave rise to the confusion which made the task of finding the proper arithmetic of our own so much the more difficult.

I have been dwelling upon the chaotic conditions of the teaching of our middle school mathematics, which reigned as late as some twenty five odd years ago. At one time it seemed to be almost hopeless to come out of this labyrinth. Now I have been using the specification of the different kinds of——more properly expressed, the different ways of treating ——elementary mathematics by such names as American, French, German, and so forth. Before proceeding further, I wish to state once more, in order to avoid misunderstanding, that I mean thereby the school mathematics *of those days* in the respective countries.

The return in 1877 of Dr. (now Baron) Kikuchi from England where he had studied mathematics for some years at Cambridge, marks the dawn of a new period, during which the school mathematics of our own type was to be remodelled and so formulated as to suit our educational organization. On his return home, he was immediately appointed the first Japanese Professor of Mathematics at the Tokio University, just then reorganized. He taught elementary mathematics as well as higher mathematics. English text-books were freely used, and even his lecture was for some time mostly given in English. Several years were to elapse before he could familiarize himself with the chaotic conditions, explained above, pertaining to the teaching of elementary mathematics in our secondary education. Thus it was only after nearly a decade has slipped away that his powerful influence came to be felt in this direction. [Cf. An article called "Mathematico-Physical Science in Japan", by my friend and colleague Professor J. Sakurai, to be found in "Fifty Years of New Japan," Vol. II, pp. 246-250.]

We owe it to Professor Kikuchi that the results of the labour of the Association for the Improvement of Geometrical Teaching founded in England in 1871, were made known here without any loss of time. The fourth edition of "the Syllabus of Plane Geometry" corresponding to Euclid Books I——VI, prepared by the Association, was published in 1885, and its Japanese translation by Professor Kikuchi was published here early in 1887. Meanwhile he was asked by the Department of Education to write a text-book of elementary geometry. We can well imagine how difficult it must have been for him to squeeze out the spare time for doing the work of compilation, of the busy hours absorbed by his function as university professor and pioneer exponent of the real progress in educational and scientific matters. However, thanks to his untiring efforts, there was published,

before many years had elapsed, his Plane Geometry which is the first Japanese work on elementary mathematics free from the tinge of translation, and which was soon followed in less than a year's time by his Solid Geometry. Indeed the appearance of Kikuchi's Geometry in 1888-9 is to be looked upon as the conspicuous milestone in the progress of the teaching of mathematics in our country, which separates the period of random adoption of foreign text-books and that of well-digested compilation of our own.

Kikuchi's Plane Geometry is based, as the author tells us in the preface, on the "Elements of Plane Geometry" published by the English Association for the Improvement of Geometrical Teaching, which, as may be noticed in passing, was translated by some one into Japanese, but differs from it in very many important respects including the treatment of parallels and the theory of proportion. His book is a very great improvement upon the "Association Geometry," at least in the sense that it adapts itself so admirably to our national character and to the environments of our educational organization. For foreigners or even for our own countrymen not in touch with the actual state of affairs in these days, it would be hard to realise the difficulties with which such a task of compilation was beset. Besides the intrinsic difficulties of producing a really improved text-book of geometry, not only most of the technical terms were to be coined anew in such a manner as to meet with general approval, but also the language itself to be used in such a book, which, as is hardly necessary to add, must ensure clearness and conciseness of definitions, brevity and perspicuity in stating axioms and enunciating theorems, and accuracy and non-redundant completeness in general. How successfully the author mastered the difficulty, is indeed a marvel in view of the peculiar and inimical nature of the Japanese language. There still remains another characteristic feature of the book

which it may be worth while to notice. Hitherto our ordinary way of writing vertically downwards in successive upright columns following one another from right to left, was used in mathematical publications. As this was found to be obviously inconvenient, there was proposed for mathematical writing a new mode of writing horizontally from left to right after the fashion of occidental writing and was actually tried in some out-of-the-way mathematical publications. For the first time, however, this mode of writing was definitively adopted in Kikuchi's Geometry, and this had the effect of establishing the usage which has been followed ever since.

Not much more than two decades have passed away since the first appearance of this first mathematical text-book of genuine Japanese compilation, and, during this interval, many, perhaps too many, text-books of geometry written in Japanese were published. Most of them owe, as a glance would at once convince any *connoisseur*, a great deal to this book, notwithstanding that their indebtedness is unjustly not even acknowledged. Indeed, for the sake of justice, it is to be much regretted that some of them are mere imitations trespassing the bounds of morally legitimate compilation. Now I would have gladly passed over all this, if not for the superfluity of books mentioned before, which seems to be a world-wide evil and is prone to give rise to similar abuses.

I have very many reasons for believing and I do believe that this book is unsurpassed by any other Japanese text-books of elementary geometry which have appeared up to this day. The position occupied by Kikuchi's Geometry here in Japan during the last two decades, may fairly and not inappropriately be compared to that held by Legendre's *Éléments de géométrie* in France during the several decades in the early part of the last century following its publication which took place towards the close of the eighteenth century. Some of the more important characteristic

features of this book will be examined circumstantially in the chapter which follows.

It was in 1888 i.e. the year in which the first Part of the Geometry above mentioned was published, that the writer of the present report was unofficially asked by the Department of Education to write text-books of arithmetic and elementary algebra. Perhaps I might have tried to compile a text-book of algebra. The task seemed to be not altogether hopeless. But the work of writing a text-book of arithmetic appeared to me to transcend my best efforts. So I have done my best by declining the request. At the same time I have made up my mind to devote any spare time I might be able to find, to contributing toward the apparently simple, but in reality extremely difficult, task of constructing the arithmetic of our own, which we *must* have sooner or later.

The first seminary courses (*kōshiukwai*) for middle school teachers, ever planned by the Department of Education, took place at the lecture hall of the Tokio Art School in Uyeno Park during the summer vacation of 1890. The subject to be reviewed was elementary mathematics, and I was asked to give a course of lectures on arithmetic. So I took that opportunity of laying before the assembly of middle school teachers the kernel of the future arithmetic of our own, which I had been thinking over. Ever since I have done my best not to lose any opportunity of making the result of my enquiry known to those who take interest in such matters and it has been freely discussed by them. The final outcome is embodied in a pamphlet entitled "Syllabus of Arithmetic and the Method of Teaching" published in 1895.

Very few words seem to have such a variety of significance as the word arithmetic, sweeping, as it does, the whole range from Gauss' *Disquisitiones Arithmeticae* to the "Arithmetic for Beginners." We have been speaking of American, English, French, and German arithmetics *of those*

days. Let us now briefly describe what they looked like or how they appeared to us at the time on having been imported into our country. In so doing, in order to avoid confusion, I shall temporarily use the words *arithmetic* and *algebra* in nearly the same sense as they are commonly used in English schools.

It would be supremely, if not insurmountably, difficult to define clearly the conventional limit between arithmetic and elementary algebra. The distinction is, of course, not essential, but is necessary for making clear what I am going to say. I shall follow in the main the delimitation as delineated in the several articles contained in the well-known old, but not antiquated, classic "Encyclopedia Metropolitana", which, I believe, is still followed to this day in England.

The prominent feature, if not the most prominent, of the English (and also American) arithmetic is that it forms a complete whole, as a matter of course, intimately related to, and yet apart from, algebra. Other distinguishing characteristics seem to be the preponderance given to *fractions* which is only natural in view of the non-decimal character of English compound numbers, a slight remnant of the duodecimal calculation which reminds us of the "scale of notation" in algebra, the tendencies of enlarging the number-domain as far as possible in mental arithmetic, a perhaps insignificant trace of the spirit to be found in olden days and embodied in calling *proportion* the *golden rule* or the *rule of three* which seems to have been occasionally put into the shade by the eruptive introduction of the so-called "unitary method," or "reduction to unity", and lastly, but not the least important, the promiscuous heaping of examples, and problems, due no doubt to the after-effect of various examinations at all stages, which makes itself felt all along the line.

The American arithmetic which was framed upon English models cannot possibly, and, in fact, does not materially,

differ from the English arithmetic, save that it is freed from the excessive treatment of fractions. One feature of the American arithmetic, which it may not be altogether superfluous to notice, is that the conventional distinction of *abstract* and *concrete* numbers is pushed to the front.

There seem to be two sorts of the conception of *ratio*. Besides the rigorously mathematical one, there is a conception of ratio which has its root in the common-sense of the educated public, and is to be found, so to speak, permeating the mass of everyday affairs. It is not to be disputed that this latter conception is somewhat vague from the mathematical point of view, but, strange to say, it is singularly not misleading, notwithstanding that our vocabulary is insufficient for defining or even explaining it. Provisionally, it may be called the *common-sense* conception, or perhaps *arithmetical* conception of ratio. It cannot be taught at one stroke, but is to be gradually developed in the course of the teaching of arithmetic, which has to do with general education. And indeed the conception is to be evolved in the juvenile mind in such a manner that it does not prove to be detrimental to learning later the strictly mathematical ratio.

An English dictionary which I happened to consult, says: "There is no intelligible difference between a ratio and a quotient of similar quantities; they are simply two modes of expression connected with different associations. But it was contrary to the old usage to speak of ratio as a quantity——a usage leading to intolerable complications. .....By mathematicians *ratio* is now conceived and spoken of as synonymous with *quotient*." Now, why it led to intolerable complications, is due not to the old usage, but to confounding the two sorts of the conception of *ratio*. It is true that there has been tendency of giving more and more importance to the mathematical conception at the expense of the common-sense conception. However, this is to be looked upon as an endeavour to suppress a really

useful conception which has taken firm root in the mind of the larger public, for the sake of convenience on the part of mathematicians. Such tendencies can only have the effect of widening the gulf which separates the teaching of arithmetic in schools and its practical applications to matters of daily concern——the complaint which we hear so often nowadays.

We have been speaking of the two sorts of the conception of *ratio*. After all, it may perchance be that we are using one and the same word *ratio* for two different ideas which are organically interwoven. In any case, the delicate shade of difference in meaning or conception belongs to the category of ideas which we can feel but not express. Sometimes it is said: The (common-sense) ratio of one number to another is the relation between the two numbers conceived through contemplating how often the former is contained in the latter. And the (mathematical) ratio of one number to another is, as the dictionary above cited says, the quotient of dividing the former by the latter. Obviously this does not cover the whole.

It is this common-sense conception of *ratio*, with which alone we have to do in arithmetic, and which gives a peculiar charm——an extremist would perchance say *mystic air*——to *proportion* defined as the equality of two ratios, and makes it such a convenient method of arithmetical computation that it is worthy of having been called, on account of its great utility and its extensive application, the *Golden Rule*. There are some who wish to eradicate *proportion* altogether, and substitute in its stead the so-called "reduction to unity." They are the people who are unable, or do not wish, to comprehend the real and beneficial significance of the common-sense ratio, and are trying, though unknowingly, to alienate the teaching of arithmetic in the general education from the ease and tangibility, with which it may be applied to the affairs of daily concern.

It seems as if, on the whole, this common-sense conception of ratio remains still dominant in English and American arithmetics. Such being the case, some may say, they are falling behind the progress of mathematics. Such extremists seem, on the other hand, not to know that they are wishing to leave nothing in this world but what will serve the convenience of mathematicians. One thing more remains to be said concerning English and American arithmetics. In them, the conception of *number* is essentially based upon *counting*, although we seldom hear of such formidable name as the Zählprincip.

William Spottiswoode, whom I have always been in the habit of holding as one of the world's greatest authorities on scientific matters bordering on worldly affairs, and who was so modest as to say in opening his presidential address to the British Association of 1878——"your choice has fallen upon one outside the sphere of professional Science"——said in the course of his address "..... What was the origin of mathematical ideas [of number and of figures]? Are they to be regarded as independent of, or dependent upon, experience? The question has been answered sometimes in one way and sometimes in another. But the absence of any satisfactory conclusion may after all be understood as implying that no answer is possible in the sense in which the question is put; or rather that there is no question at all in the matter, except as to the history of actual facts. And even if we distinguish, as we certainly should, between the origin of ideas in individual and their origin in a nation or mankind, we should still come to the same conclusion. ....." Finding myself in substantial accord with his thought and discovering that he has said just what I longed for to say, I wished so much to quote at greater length than is done in the above. The reader is referred to the Report pp. 28-9. The result of profound deliberations as applied to the teaching of arithmetic in middle schools,

seems to converge toward the conclusion: The less is said of the conception of number, so much the better it will be not only in teaching arithmetic to boys, but also and particularly in discussing pedagogically the methods of the teaching of arithmetic.

Let us now say a few words concerning the German arithmetic. If the *ensemble* of *Gemeine Arithmetik*, now and then called *Rechnen* simply, and *Allgemeine Arithmetik*, sometimes named *Arithmetik* without any qualifying prefix, be taken for the German arithmetic, then it is of much wider scope than the arithmetic we have been speaking of. It may be compared to arithmetic combined with no small portion of algebra. It is interesting to observe in passing that the name *Arithmetica Universalis*, of which we do not hear nowadays in the country of Newton, is still preserved to this day in the country of his rival. One of the noteworthy features of the German arithmetic is the comparatively small number of exercises and problems to be worked out by pupils, which are carefully selected and methodically arranged. This is no doubt mainly due to the scarcity, if not the absence, of *external* examinations, such as are too often required in our own country in passing from one kind of schools to the next higher, which, in turn, is due to the excellent educational organization approaching almost to perfection.

Next, as to the French arithmetic, it seems to lie intermediate between the German and the English or American arithmetic, in the aspect that, in it, notably towards the end, letters are to a certain extent almost freely used. The introduction of number into the very beginning of arithmetic can not possibly take place except by counting, and such is the case even in the French arithmetic; but as soon as *fraction* is reached, the idea of *grandeur* and even of *grandeurs continues* is introduced, and the conception of number is derived anew from the com-

parison of *grandeurs* i.e. measuring. For an outsider, it seems only natural that the simultaneous derivation of number by counting and measuring can only have the result of making arithmetic for beginners rather confusing. I have been always in the habit of considering the conventional introduction of *abstract numbers* and *concrete numbers* as a happy means of passing over——we may even say slurring over——the difficulties most likely to be experienced by beginners in seeing numbers derived by counting applied to the result of measuring, or, otherwise viewed, the number derived by counting identified with the number got from measuring. It seems rather odd that in some French text-books of arithmetic, one finds, though rarely, along with *nombres* and *grandeurs* such words as *nombres abstraits* and *nombres concrets*.

The French arithmetic penetrates considerably deeper than other arithmetics into the *properties of numbers*, i.e. the rudiment of the abstruse (for non-mathematical laity) *theory of numbers*. It seems, nowhere else do we meet with such a large number of problems which have to do with *divisibility*. The subtle idea——or it may perhaps be called tendency ——which it is vain to attempt to express in words, but is found embosomed in using almost exclusively one and the same fractional form (colon is used very rarely) for indicating division, and for writing fraction equally with ratio, seems to be a distinguishing characteristic of this arithmetic. Another characteristic feature is, fraction and ratio treated so much alike that the question, which may suggest itself to a beginner, seems legitimate——Why are two different names *fraction* and *ratio* given to one and the same thing? I was told that much has changed since, although a glance at the 1873 edition of Briot's *Éléments d'Arithmétique* and the 1907 edition of Borel's *Arithmétique* which happened to be near at hand, has not fully convinced me of it.

There is a Japanese text-book of arithmetic in two

volumes written by my friend and colleague, Professor Terao of the College of Science, Tokio Imperial University, the first edition of which was published in 1888. It is one of the very small number of text-books of elementary mathematics written in Japanese, which are free from the shade of translation, and are the product of well-digested compilation. It is a book which will become never superannuated. The French arithmetic which has become in course of time to be known by the name of "arithmetic of the theoretical type," was in fashion at one time, although it has left but little trace on our own arithmetic of the present day. This book of Professor Terao will ever remain the worthy monument of the so-called arithmetic of the theoretical type.

Before closing this chapter, I wish to be allowed to say once more, even at the risk of redundant tautology, that I have been speaking of the English or American, French, and German arithmetics *of those days*, and indeed, as they *appeared after having been introduced into our country*. There is a chance that the text-books mentioned above might not be typical of those of the country which they were meant to represent. Again the interval which separates to-day from *those days*, might have witnessed some radical changes in the teaching of arithmetic in these countries. For all that, the above account of some characteristic features of these different arithmetics was deemed necessary, as therein are to be found some of the materials, with which we had to build up the arithmetic of our own.

CHAPTER VI

THE TEACHING OF MATHEMATICS IN MIDDLE SCHOOLS (*continued*)

However desirable it might be to make the teaching in middle schools as pratical as possible, the off-hand utilitarianism must not reign supreme in the teaching of elementary mathematics. The aim of mathematical instruction in secondary schools is to be found in the official statements of different countries, which are very much alike and so need not be recapitulated here. The more indirect aim of the teaching of mathematics as a part of the general education should be directed toward developing that mental faculty whose distinguishing traits are economy and exactness——an economy without stint, and an exactness which involves the minutest attention to the smallest of details. Again mathematics must not fall behind other subjects in paying due attention to the cultivation of the mother tongue. Particularly, to use a mathematical phrase, the *necessary and sufficient* way of thinking and expressing in words, which need not be dry and monotonous, should be practised and encouraged.

We now go on to notice some of the more characteristic features of arithmetic, algebra, geometry, and trigonometry, as taught in our middle schools.

Arithmetic

Although, following general usage, arithmetic is classed as a branch of mathematics, it is something more, or, otherwise viewed, perhaps less than a mere branch of mathematics. That it is not a branch of mathematics in the sense algebra or geometry is such, hardly requires mentioning. However, it seems desirable that this point of view regarding arithmetic should be pushed to the front. Considerations which have their origin in the purely mathematical standpoint of view should not exercise undue influence on the teaching of arithmetic in secondary schools.

There are certain kinds of knowledge indispensable to our daily need and inseparably connected with the object of the general education, which are more becoming to be included within arithmetic than in any other subjects. Such are, it need hardly be told, the knowledge concerning weights and measures, time, money and currencies, customary conventions regarding the calculation of interest, and so forth. Only, in our arithmetic, they are not accessories, but, as was said before, are looked upon as forming an essential part. In fact, we go a little deeper into such topics than is usual in the arithmetic of other countries. Besides, there are somethings which, if not non-mathematical, are not exactly mathematical, and which find in arithmetic their home at once so natural and befitting. A typical example is to be found in the common-sense conception of ratio spoken of in the preceding chapter.

The history, though comparatively brief, of the reform movement in the teaching of arithmetic here in Japan shows a series of fluctuations in shifting emphasis from one point to the other, which may recall the change of fashion in worldly vulgarities. In very many things, fashion or mode comes and goes away, repeating some sort of cycles and

returning to nearly the same phase in more or less regular intervals. In most cases, the existence of fashion finds its justification in the human weakness of becoming simply tired of things old and welcoming any change whatever good or bad. So it happens with the teaching of arithmetic. After all much of the present and the past will be found the best for the future. Indeed arithmetic contains matters which can be traced unfailingly back to the days of primitive barter; and they are just as useful to-day as they were then. When all is said, the Golden Rule may not be a thing to be despised.

In educational matters, no reform can take place all at once. Conservatism leading to stability in such matters, has at least the negative virtue of dispensing with the waste of energy involved in revising over and over again without awaiting the ripening of the preceding revision. The first requisite of a really efficient revisionary measure is the possibility coupled with the great probability of its remaining stable for the length of time necessary for its completion. In the teaching of arithmetic, there is, unfortunately, a category of reform proposals which are as often suggested as they are dismissed——indeed almost as regularly as the ebb and flow of the tide or as the sun sinking in the west is sure to rise from the east. Such are, for instance, the mixing up of counting and measuring in giving the first notion of numbers, complementary addition, suppressing proportion and replacing it by "reduction to unity," and so forth. Another example is to be found in the recommendation to banish indiscriminately from arithmetic whatever presents the slightest difficulty which is very often only apparent——saying: "it is better to teach it in algebra."

Complementary addition, sometimes called *Oesterreichische Subtraction*, is a process of subtraction which is based upon adding the complement (from 10) of a digit instead

of subtracting the digit. Such an idea may cross the mind of somebody. There is nothing to be said against this. But it does harm when it appears to him as a great discovery destined to revolutionise the very fabric of arithmetic by reducing the *four* rules transmitted from time immemorial to *three*. At once he becomes the staunch advocate of the reform movement in this direction. But we can never make an end of subtraction in our daily affairs, and the arithmetical operation of subtraction should harmonize with the mental process of subtracting as far as possible; this can never be sacrificed for the sake of the questionable convenience of a prejudiced few. Similar remarks apply to the reform proposals belonging to the same category. To the student of human mind, it would be perhaps interesting to observe that such a zealous reformist is usually found among the authors of magazine articles hastily written and of text-books produced in one night.

Now, reform proposals as the above are found always and somewhere, so to say, floating in the air. That cannot be helped; and after all, they do no great harm as long as they remain floating in the air. It is only when they are casually gripped in the flying cogs of the bureaucratic machinery that they do real harm. By chance, such a reform proposal may have the luck of catching the fancy of an official, while officialdom is diligently grinding the bureaucratic weapons——industriously heaping up informations while smothering the life of really beneficial reform movement. Now even bureaucracy is powerless to carry through an absurd reform of this kind; but with all its prestige it can and sometimes does dictate such an innovation which infallibly leads to inevitable confusion. Thus it is necessary to meet half way any such proposal as is contrary to the dictates of common sense and to take an aggressive attitude against it rather than the defensive, so that it does not come within reach of bureaucratic hearing.

The preliminary task of any genuine improvement must, thus, consist in cataloguing all sorts of mock reforms which are likely to be proposed now and then, and in banishing them forever so that they can no longer disturb the stability so essential to real progress in the teaching of arithmetic.

To the peculiarities of our educational organization is due the break of continuity between the elementary school education and the middle school education, which, taken all round, has both its advantages as well as disadvantages. However desirable it might be to bridge over this gap concerning the teaching of arithmetic, we have to content ourselves in this respect with the available best under the existing circumstances. Now, arithmetic is taught, so to speak, lingeringly and necessarily so during all the six years of the elementary school course. For the first time, arithmetic is taught systematically in the middle school. Not only because of the inherent difficulties but also because of its having been already taught in the elementary school, arithmetic is the most difficult subject to be dealt with in the middle school, notwithstanding that it is not seldom assigned to the least experienced of all the teachers of mathematics in a school.

In giving the first notion of number in the middle school arithmetic, which must be based on counting, the less said the better it is. The same applies to the explanation of the four rules. It seems to be a mistake to try to make pupils understand such matters by explaining them. The proper course to be pursued appears to consist in leading them on so that they become proficient in calculations of daily need, and gradually and unawares become cognizant of the correctness of the principle underlying these calculations. I do not even wish to use the word "intuition," although some might suggest that what I have

been speaking may be covered by some such phrase as "progressive intuition." In this connection I cannot do better than quote a few words of Arthur Cayley from his presidential address to the British Association of 1883, which follows:

"In fact, whatever difficulty be raisable as to geometry, it seems to me that no similar difficulty applies to arithmetic; mathematician or not, we have each of us, in its most abstract form, the idea of a number; we can each of us appreciate the truth of a proposition in regard to numbers; and we cannot but see that a truth in regard to numbers is something different in kind from an experimental truth generalised from experience. Compare, for instance, the proposition that, the sun having already risen so many times, will rise to-morrow, and the next day, and the day after that, and so on; and the proposition that even and odd numbers succeed each other alternately *ad infinitum*; the latter at least seems to have the characters of universality and necessity."

After all, there is perhaps no real difficulty. Difficulty manifests itself only as soon as we try to explain something which cannot be explained. Inquisitiveness is the soul and source of mathematical progress; but here in the teaching of arithmetic, inquisitiveness, especially on the part of teachers, must be confined within its proper limits. An axiom is a self-evident truth. That something is seen to be self-evident may depend upon the intellectual development of pupils. And we see no reason why we cannot wait till it becomes axiomatic to them. Here as well as throughout the whole of the middle school arithmetic, the so-called *heuristic* method in its widest sense should reign supreme.

We have already noticed a few characteristic features of our arithmetic in connection with the teaching of arithmetic in elementary schools (Chap. III). Moreover, the time at

my disposal for finishing this report being so short, I have to content myself with giving categorically some of the salient characteristics of our middle school arithmetic, however much I may wish to say something about each of the topics enumerated below.

Decimals introduced early as an extension of the denary (decimal) system.

Concrete number introduced *conventionally* as a number coupled with the name of the thing to which it is applied. [All numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned. The phrase "concrete number" is really meaningless. Things numbered are concrete, but the number is abstract].

Concrete numbers or rather conventional conception of concrete number freely and conveniently made use of.

Compound number which has the same claim of being called *number* as concrete number.

Facts known by the name of "useful knowledge pertaining to calculation" given more in detail than in other arithmetics.

Ratio explained as the *relation* between two numbers conceived through contemplating how often the second number is contained in the first——the quotient of the first number by the second being called the *value* of the ratio.

Proportion defined as the equality of two ratios, which is equivalent to the equality of the values of two ratios.

Use of Proportion encouraged as a very convenient means of calculation and solving problems.

Facts connected with ratage (percentage) and interest more fully given than in other arithmetics.

Square root and cube root included without shifting them to algebra.

Easy mensuration included and distributed here and there.

Fractions treated very slightly.

The so-called "properties of numbers" touched very lightly.

A few words may be said concerning proportion. In the olden days of "Three-Thousand-Problems," it was customary to enumerate as many kinds of proportion as possible; for example, direct proportion, inverse proportion, simple proportion, compound proportion, distributive proportion, etc. Even chain rule was named chain proportion, and alligation, mixture proportion. To each kind of proportion was associated a sort of almost mechanical rule for working it out. When a problem was given to a pupil to be solved, he had to hit upon——perhaps guess——the particular kind of proportion to be used, and thenceforth to solve it by blindly following the prescribed rule. This prejudicial habit which lingered for some time before it became obsolete, is fortunately now entirely eradicated.

As regards square root and cube root, it has often been proposed to shift them to algebra. The reason given for doing this, has invariably been "because we cannot explain the principle upon which the process of extraction is based." Now there is a certain kind of argument or reasoning which has no real ground and yet is so readily accepted by many people. It seems to appeal to the weakest point of human weakness which economises mental exertion as much as possible in the presence of an apparently plausible explanation, and so passes on undisputed without its validity being even questioned. The above argument for shifting extraction of roots to algebra is just of this sort. Algebraic symbols are convenient in an analogous sense as stenography is convenient to a stenographer. Algebra explains in an algebraic way but not so as to appeal to the common-sense understanding. To explain the ordinary processes of multiplication and division in the sense some people wish to see the extraction of square root and cube root explained, it

would equally require algebra. Why do they not propose to shift ordinary multiplication and division into algebra?

It is true that boys very soon forget the actual processes of the extraction of roots. But that is of very little consequence. How many out of millions of the larger public will ever have an occasion of extracting square root, not to say cube root? Even those who go into the professions requiring such calculation do the same usually by means of logarithmic tables. Now, in the circumstances, it is pertinent to ask, whether it is at all necessary to teach square root and cube root, no matter whether in arithmetic or in algebra. Without any further discussion, let the teaching of square root and cube root within arithmetic be told as plainly as possible. Let us speak of square root, as the same remarks apply to cube root. Boys should know how to extract square root for a time just after he has learned how to do it. Later on he need only possess the potentiality of recovering his knowledge, whenever there is a necessity, by referring to his text-book. The ordinary process of extracting square root is based upon the usual scheme of writing down the square of the sum of two numbers, which can be fully explained by one or two illustrative numerical examples. The simultaneous comprehension, i.e. comprehending all in one thought, of the various different steps and their interrelations would far surpass the understanding of even a comparatively gifted boy, not to say of an *average* boy, who has completed his middle school course of algebra. Thus no real advantage accrues from shifting square root from arithmetic into algebra. People often say: "this or that can be explained by algebra." In most cases, however, the explanation itself is algebraical which may be sufficient for a teacher but not for boys. Very often difficulties are concealed or pushed down to lie hidden beneath the surface, instead of being cleared away, at least, in so far as appeal is made to the comprehension based upon the common-sense of

an *average* boy.

As to the desirability of retaining square root in arithmetic, there is another consideration which even outweighs that given above. Exercises in numerical calculation consisting of very many steps and involving all the four rules, are much to be desired. At the same time, we must not lose sight of the inevitable fact that boys become tired of doing the same sort of dry computation over and over again. Here what I have elsewhere named "variety in uniformity otherwise called monotony" is the desideratum. Now extraction of square root furnishes an object which just fits such a purpose. Again, broadly speaking, we may distinguish two sorts of means, with which arithmetical training is being carried out——one sort, usually covered by the name *exercise*, requiring mere numerical calculation without any special object, and the other which may be called *problems* in a specific sense, requiring, beside calculation, more or less thinking, or digestion of some knowledge pertaining to daily affairs. And it needs hardly mentioning that in the proper distribution of these two sorts of means is to be found the key which opens the door leading to the full comprehension of arithmetic. Toward the end of arithmetic as it is usually taught in middle schools, there is to be noticed a tendency toward the condensation of matters of the nature of *problems*. That some slight rarefaction should take place by the introduction of exercises on the extraction of square root, is just what is most desired.

Throughout the whole course of the teaching of arithmetic, verification, technically called test or proof, should be encouraged to the utmost. It would be well nigh impossible to overestimate the importance of the cultivation of those habits of patient work and self-reliance which would result from pupils' being always required to verify the accuracy of their own calculation,——the only test they will have to

depend upon in all the computations in after life. More than that, the work of verifying a result is very often of quite as much value to them as the work of going through the direct process; the verification may make clear some part or the whole of an operation that before was entangled and obscure. The excuse, too often put forward, that there is no time, ought not to be allowed to obstruct the way to this end.

We hear it often said——indeed I myself said so somewhere in this report——that our arithmetic is framed on American models. But, in reality, our arithmetic has been constructed, based upon the considerations, of which the main traits have been delineated in the above. So, after all, it may perhaps be nearer the truth to say that the arithmetic of our own thus constructed very much resembles that of the American type. To sum up, our arithmetic is to be looked upon as something *not quite mathematical*, in the sense exemplified by the conventional introduction of concrete number and the propitious adoption of the common-sense conception of ratio. That it is classed, following the established usage, as a branch of school mathematics, is merely a matter of form. In fact, it occupies an intermediate position between some common-sense knowledge pertaining to the affairs of everyday life and a purely mathematical subject such as algebra or geometry. On this very account, it is highly desirable that arithmetic should form a separate entirety apart from other branches of mathematics, in particular, from algebra. By thus defining the character and the scope of arithmetic, the object of teaching it in middle schools will be more thoroughly attained than by doing otherwise.

The work of compiling a text-book of middle school arithmetic on the lines depicted in the above, was completed, after much labour which lasted several years, by the writer of the present report. It was published in 1896 and its

abridged edition in 1899.

It remains to add a few more words concerning arithmetic. It was the middle school arithmetic which was first framed, and the elementary school arithmetic was constructed by following it too servilely. Thus there is still left the task of giving to the latter such distinguishing characteristics as would harmonize with the object of teaching arithmetic in elementary schools.

Algebra

Unlike arithmetic, algebra or geometry is to be looked upon as a purely mathematical subject which should be taught and learned for its own sake. Algebra or geometry is a subject capable of wide application, but not in a direct way and in the sense in which arithmetic is applied to affairs of everyday need. Nothing can do more harm than off-hand utilitarianism in discussing the proper method of teaching algebra or geometry in middle schools. In this respect too, algebra or geometry is very much unlike arithmetic. Thus, in the very spirit which should pervade the teaching of arithmetic and of algebra, there is a clear distinction which must be strictly observed in order to make the teaching of these subjects really beneficial. And as a natural consequence, arithmetic was to be so constructed as to form a distinct whole apart from algebra, and for this purpose, even a certain amount of repetition which is however often only apparent, was to be taken into the bargain.

Very often the phrase "it can be explained by algebra" comes out to be meaningless, just as one who says "I will do it to-morrow" seldom does so——which is a very bad habit. In fact, algebra does not give explanation in the sense some people use the word "explanation;" on the contrary, it very often gives the method of evading or circumventing such an explanation. An example will make this clear.

In our arithmetic, there are problems belonging to the so-called category of "crane-turtle" problems——a name perchance derived from the mathematics of the old Japanese school. Such a problem is e.g. the following: "There are altogether twenty cranes and turtles, and the total number of legs is fifty-four. How many cranes and turtles each are there?" The usual arithmetical solution runs in outline along the following track: If all were cranes, there will be

40 legs instead of 54, giving rise to the difference of 14 in the total number of legs. Now, every time we replace a crane by a turtle, the number of legs will be increased by 2. Hence, in order to make up for this difference, we have to replace as many cranes by the same number of turtles as 2 is contained in 14. Dividing 14 by 2, we get 7 for the required number of turtles, and thence 13 for the number of cranes. Thus every process is clearly explained in the sense that the explanation appeals to the common-sense comprehension of pupils. For them, it may be somewhat difficult to find the track; but, once on the track, what follows is a plain sailing. The algebraic solution of the problem is about as follows. Denoting the number of cranes by x and the number of turtles by y, x and y are put into a mill called "solution of linear simultaneous equations," then follows the grinding of the mill, and out comes the required answer. Now all this is at once simple and easy, but it is so because of its dispensing with explanations. Indeed, herein lies the life and beauty of the operations in letters or symbols.

It is to be much regretted that, here in Japan, the distinction which I have tried to elucidate in the above, but may not have succeeded in making clear, is too often mixed up by those who wish to reform the teaching of arithmetic and algebra in middle schools. To this confounding are to be traced some of the unsound reform measures which have been proposed from time to time and occasionally even adopted.

No branch of mathematics seems to have enjoyed such a wide application as algebra. We need hardly mention its application to physical sciences such as physics and chemistry, to which we may add the medical sciences of theoretical trend, political economy, insurance, statistics and even psychics. Not only the algebraic formulae but also the algebraic

way of thinking in symbols have been applied to affairs lying not much farther than those of everyday need.

In the foregoing, it was said that we *must* have the arithmetic of our own. This ought not to give a false impression as if we are going to have also the algebra of our own. Nothing can be farther from the truth than that. Algebra cannot possibly differ much in different countries except in regard to the scope allotted to the particular kind of schools.

Ever since we gave up using foreign text-books of algebra in the original as well as in translation, the predominant feature of algebra as taught in our middle schools has been the unreserved adoption of the principle of the *permanence of form*, already suggested by George Peacock as early as some time previous to the middle of the last century, and so fitly named by Hermann Hankel "*das hodegetische Princip der Permanenz der formalen Gesetze.*" It need hardly be said that we never even mention the name of this principle in actual teaching. The successive new numbers——the negative, the fraction, irrational and complex numbers——are introduced, keeping this principle in view on the part of the teacher but dictating dogmatically the conventions regarding them to the pupil.

In days of old, much confusion arose out of trying to explain matters which are, in reality, no more than mere conventions. For example, in order to explain the "law of signs" of the product, the definition of multiplication was so modified that it runs: "To multiply a number by a second is to do to the first what is done to unity to obtain the second." Similarly, whenever and wherever we met with any difficulty, definitions and explanations were so modified as to suit the particular circumstances, without thinking of the confusion, to which we shall be led by retracing the

metamorphosis of these definitions and explanations. To the similar cause is due the change of the names *greatest common divisor* and *least common multiple* to *highest common factor* and *lowest common multiple* respectively. All the causes which are destined to give rise to such confusion, were banished for ever by the bold acceptance of the *Princip der Permanenz der formalen Gesetze*. The beginning of algebra as taught in our middle schools has been thus made as simple and easy as it can possibly be.

Let us now notice a few of the salient features of algebra as taught in our middle schools:

The fundamental laws to be simply noticed.

Conventional matters to be given as such and not to be explained.

Negative numbers to be introduced at an early stage.

Simple equation to be taught not in one aggregate—— easy examples to be given as early as possible.

Methods used in solving equations occasionally to be referred to the first principles on which they are based.

Roots of equations to be always verified for their own sake as well as for serving as an exercise in substituting numbers in place of letters.

Numerical evaluation of algebraic expressions to be associated with some definite object of evaluating as far as possible.

In solving fractional equations, the steps by which the denominators are struck out to be always kept in hand, so that they can be retraced.

Surds to be evaluated numerically.

Imaginary and complex quantities to be avoided as far as possible.

Graphs to be introduced at a convenient stage and made use of afterwards occasionally.

In giving graphs, we have to confine ourselves to mere plotting and a few simple and easy applications. The thirst for something comparatively new notwithstanding that there is some truth in "*Das Gute nicht neu, das Neue nicht gut*", and the vague impression that something must be done for the sake of algebra keeping company with geometry in being reformed, often lead to going somewhat deeper into graphs than would be at all necessary or desirable in middle schools. Let those reformists, to whom the introduction of graphs into school algebra may appear as the greatest discovery of recent times, be reminded of the difficulties which some of them might have had in learning the beginning of analytical geometry at an age considerably in advance of the average age of middle school boys.

There seems to be one direction, in which real improvement in the teaching of algebra may take place. In algebra, numbers are represented by letters which appear to gifted ones as general numbers, but to *average* ones as mere symbols. And algebra treats of operations in symbols. Thus, considered from the standpoint of mental discipline, the teaching of algebra is liable to become, on the part of *average* boys, parasitic or even hyperparasitic. So far as this tendency is due to the very nature of the subject, it cannot be helped. But there seems to be one way, along which much may be achieved in removing this evil.

As we go on higher and higher in the study of mathematics, there takes place more and more what we may perhaps call *condensation of wording* which is only natural. Somehow, condensed wording which came into use in connection with higher mathematics, seems to have crept in unawares into middle school algebra——due perhaps to some such circumstances as an author wishing to give the appearance of

novelty to his new text-book of algebra. An example or two may make clear what we wish to say.

Instead of saying, "less or equal," we can just as well say, "not greater." The latter is a wording which is obviously more condensed than the former. This is merely intended for illustrating what I have named condensation of wording and ought not to convey a false impression that a middle school boy will have difficulties in understanding the phrase, "not greater." Again it is often said, "an identity is an equation in which the two sides are equal whatever numbers the letters represent." Here a necessary and sufficient condition that an equation should be an identity is taken for its definition, and, as we know, this is very convenient for the purpose of higher mathematics, but surely it does not readily appeal to the comprehension of beginners. The more direct way of stating something like, "two different forms of in reality one and the same expression put equal to one another is called an identity," is certainly more intelligible and is sufficient for all the purposes of elementary algebra. Here again, in actual teaching, it would perhaps be of not much consequence, which of the two definitions we adopt. It is intended merely for illustrating the intrusion seldom noticed, into the very rudiments of algebra, of ideas and the corresponding wording necessarily condensed, which find their proper abode in more advanced algebra.

It is a well-known fact that beginners have great difficulty of comprehending the well-known rule for finding the greatest common divisor. Now it is usual to begin by saying: "Let A and B denote the two expressions." This seems to be at once simple and easy. Nevertheless, on the part of an *average* boy or one a little below the *average*, the difficulty begins at this very beginning. It is true that the explanation or statement of the general rule is usually preceded by a few illustrative examples. But even this is of but

little avail. Not long ago boys were for the first time taught that a letter represents *any* number. A little later, they come across an aggregate of letters or *general* numbers, which is called an expression or a polynomial. So far, their comprehension may keep pace with the progress. But now they are told that A or B represents any such expression or the *general* polynomial, and indeed in one breath which may be interpreted as implying that all this is so simple as not to require any elucidation or even halting. The worst feature is that both the teacher and the pupil are not conscious of the presence of the difficulty which is passed over in silence, as the pupil is unable to locate it precisely, and so does not ask his teacher for an explanation. I do not mean to say that the representation of any expression by a single letter is a matter inaccessible to the understanding of even a boy below the *average*. What I wish to insist upon, is that teachers should do more in the way of reading the mind of pupils. A suggestion may find its place here. Let the teacher pick out a few pupils out of his class, one of about the *average* standing in the subject and others representing respectively the different grades below the *average*. By giving them a *tête-à-tête* instruction i.e. instructing individually by question and answer in catechetic form, and then examining them *viva voce*, it may be possible to locate the difficulties they are really encountering. The knowledge thus gained on the part of the teacher may be utilised for the benefit of the whole class.

Particularly, in the teaching of algebra, the difficulties experienced by boys do not occur in one lump. They are the aggregate effect of minor obscurities occurring here and there and heaped one upon the other, such as the condensation of wording and the abrupt introduction of thinking in a manner but too common in advanced algebra, as has been exemplified in the above. Difficulties on the part of

pupils, incapable of locating them, are the worst of the kind. If a difficulty has been approximately located, there has already been made a big step towards clearing it up, and it vanishes altogether as soon as its exact spot will be found out. In all subjects, especially in mathematics, and more particularly in algebra, there seems to be ample room for improvement in the method of teaching in this direction of giving more thought and importance to the minor difficulties, with which the pupils of average standing are beset unawares. To the neglect of such caution are to be attributed some of the causes, not the least important, of the current prejudices against mathematics to be found among modern schoolboys.

Geometry

Here in Japan, as, I believe, elsewhere, there have been raised against the difficulties of teaching and learning geometry complaints as numerous as they are varied. And there are so many, perhaps too many, people who wish to have a voice in discussing the improvement of the teaching of geometry in middle schools. Indeed it is tempting to play upon the surface of the question; but to handle acceptably so difficult a problem, requires a sound knowledge of really essential fundamentals and a thorough digestion of the conditions by which it is circumscribed.

Nothing can do more harm to the cause of really improving the teaching of geometry in middle schools than the idolization of impromptu utility. Geometry, as taught and as can possibly be taught in schools, can be of little use to any one in the business of life. We appeal to every one who has been through the school routine, whether this be not the case. Even for those who continue their mathematical studies into the region covered by the name of higher mathematics, it is the mental discipline in geometrical reasoning which is so often of use to them in their after studies. We again appeal to every one of them, whether the knowledge of geometrical facts they may have to apply in their study of higher mathematics be not incomparably insignificant. Alike for those who have nothing to do with mathematics in their after life and for those who are going into professions requiring the knowledge of higher mathematics, geometry in the middle school curriculum is indispensable as a means of training them in logical reasoning by making *conventional* use of geometrical figures, just at the stage when the intellectual development of boys is liable to take such a turn as to seek ease by going astray from the path of such training. If, to meet the argument

that boys find it so hard, it be proposed to evade the real and essential parts of geometry, for the purpose of making it appear that something *called* geometry has been learned, we do believe that it would be far better to strike off the word geometry from the list of subjects taught in the middle school. If some special points in geometry be found too difficult for the *average* boy, they may be deferred till he gets a little older; in the mean time, they may be taken for granted for a while, whereby such assumptions must be always accompanied by the explicit and unequivocal statement that they will be proved afterwards. If geometry can be made simpler, by all means, let it be made as simple and as easy as possible, so long as it does not endanger the very existence of geometry as a subject destined for training in logical reasoning. The difficulty of teaching geometry should be reduced to a minimum; but then we must be prepared to face, but not evade, this minimum difficulty. Surely, training in logical reasoning and the consequent development of the reasoning faculty is a requisite of general education. And where is such training to be found except in the teaching of geometry? Indeed, geometry as a subject of liberal culture is to be looked upon as logic made simpler and easier by the conventional use of point, line, and so forth instead of reasoning in the abstract.

There are some people who wish to see geometry and arithmetic correlated to one another through the intervention of mensuration. Seemingly plausible as this may appear, in reality, it involves something like self-contradiction. In mensuration, so far as we are concerned with rectilinear figures, we need only know how to find the area of a triangle. Now any one who knows how to find the area of a rectangle, ought not to have the slightest difficulty in finding the area of a triangle. By appealing to the common-sense or intuitive understanding, any triangle may be divided into two parts, by letting fall a perpendicular from

the vertex upon the base, and then, further on, there remains no more than that two equal right-angled triangles may be so placed as to form a rectangle. Again, let it be required to find something in mensuration more difficult than the area of a rectilinear figure. An engineer who does not happen to know by heart the needed formula——even if he does, for safety's sake——he immediately refers to his invaluable pocket-guide, the "Practical Engineer." Why this privilege, granted to an engineer, of referring to a ready-made formula should be denied to a boy learning mensuration, is indeed difficult to see. Arithmetic should appeal to the common-sense comprehension of boys, in order that its teaching comes to be *practically useful* in the most ordinary sense of the words. Geometry has to do with the training of strict and exact reasoning. The aim of teaching these two subjects is thus entirely different and, in a certain aspect, even contradictory. In fact, it cannot be otherwise. Arithmetic and geometry should be kept apart as far as possible, in order that boys do not get confused and confounded in the very spirit, in which the respective subjects should be taught and learned. Among various subjects taught in the middle school, there are some, perhaps a great many, which may be made correlated; and, indeed, very often it is desirable to have several subjects mutually related. But there are exceptions as in all other cases. Even sugar and salt can be mixed together, but oil and water never.

In the preceding, I have ventured to give a bird's-eye view of discussions and controversies which have been going on concerning the teaching of geometry, and the focus towards which their results seem to converge. Now there seems to be one point which has hitherto escaped general notice. Boys of the present day seem, generally speaking, to be not so much in earnest in learning as those of some thirty

years ago, i.e. of the days of *Ei-Kwan-Sū*,* as the phrase goes, when these alone formed the chief subjects of study. All this is to be attributed to the ever mischievous working of the superfluity of publications. There are nowadays all sorts of magazines for young people, such as magazines for boys, for girls, and even for infants. These publications profess to give the reader useful information, while he or she is being amused. And this seems to appeal so acceptably alike to the young people and to their parents that the number of such publications is increasing year after year. Now young people read only those portions of such magazines as happen to catch their fancy. No efforts are spared on the part of the publishers to make their magazines as agreeable as possible to the young folks, going not seldom even to the length of appealing to their weaknesses. At the same time that young people get some hazy knowledge of profusion of topic, they are unawares habituated to looking for the acquisition of knowledge in a leisurely and easy-going way. Virtues are so hard to implant into the juvenile mind, but vices, particularly those whose worst features lie hidden behind the screen of apparent usefulness, are readily rooted in one unlucky night. The moment a boy becomes fond of such a magazine, to him is lost for ever that earnestness which is of vital importance not only in his school work but in all the realities of his after-life. And sometimes it ends in his becoming himself a writer of such magazine articles, thus propagating the evil wider and wider into the next generation. What is taught in the school appears to him to be very much like what he dimly remembers as having seen somewhere in magazines——only with this difference that it is not so complimentary and attractive as the magazine articles. It is a sad fact that

* "Ei-Kwan-Sū" means English, the classic Chinese, and mathematics.

to some magazine articles are prefixed the authoritative names of some well-known savants like that of a university professor, whom schoolboys hold far more in esteem than the teacher they are facing. To the same cause may be ultimately traced that nowadays boys continually talk of school work as something to be evaded and teachers as men to be outwitted. Just a bit of pigment will give a fair and definite impression on pure white, but any amount of colouring substance would be of little avail in giving a well-defined colour to a surface already tinted so often with all sorts of promiscuous hues.

Fortunately, the evil just spoken of, is, comparatively speaking, not of very great consequence in the elementary education as well as in that which lies above the middle school education. Intermediate between them, it is felt most keenly in middle schools. And among all the subjects taught in middle schools, geometry is just the one which suffers most from this evil. Considered from the same point of view, it is clear that the suggestion, sometimes made, to lessen the difficulties of geometry by introducing a little of so-called mathematical recreation, can only do harm. As in all other cases, such a seeming remedy augments the very evil, which it proposes to counteract. Once boys find fascination in such an amusement, so much attention will have been detracted from the proper direction, in which it should have been applied. They would even covet such recreation ever more, and would become disposed to turn a deaf ear to anything unattended by such amusement. It is the concentration of mind both on the part of the teacher as well as on the part of the pupil, leading to the unsought consciousness of both breathing the same air, that is absolutely indispensable to the really efficient teaching of geometry. On this very account geometry is a difficult subject, and there is no way of escape except by eliminating it altogether from the list of subjects taught in the middle school.

The excessive number of magazines for young folks to be seen everywhere nowadays, does mischief of such magnitude as would far surpass the imagination of those who have not paid special attention to this new form of social evil. They do mischief in the way of unpreparing boys for receiving middle school education as well as by disturbing them all the time while they are receiving such education. Quite recently a cause of the grievances against the ineffectuality of the middle school education was traced to the system of examinations. Now, however great the evils of examinations are, they are not to be compared with those of boys coming into contact with the so-called magazines for young people. Indeed the very necessity of such examinations is partly due to this cause, they being used as a means of recovering just a little of that earnestness which boys have lost by dawdling over such magazines. Absent-mindedness, lack of harmoniously taking to each other of those who teach and those who are taught, and the like grievances, of which we so often hear nowadays, may all be in the end traced to this evil covered by the fine name of supplementing the school education. It is by no means an exaggeration to say that more than half of the school work is thus made fruitless.

It is very often desired that a first introduction to geometry should not be formal but experimental, with the use of instruments and numerical measurements and calculations. There is nothing to be said against this, provided it does not lose the character of being preliminary to the real study of geometry and that special emphasis is laid upon the clear distinction between such preparatory steps and geometry properly to be so called which is the science of reasoning. Again, at the first stage of the teaching of geometry, teachers should be on the look out for individual boys who

exhibit backwardness or any mental peculiarity, and should bring them under special care. The class work in geometry is very often disturbed by a few individual boys who may happen to be bewildered by some difficulties and ask meaningless questions paralyzing the work of the whole. The time thus spent is not a waste; on the contrary, it is more than amply repaid by the smooth working of later lessons. Even the anxiety about *no time* afterwards ought not to stand as an obstacle on the part of teachers for acting as just recommended.

As was said before, geometry as taught in our middle school, is essentially that which is represented by the book of Baron Kikuchi. His text-book consists of two parts:— "the Elements of Plane Geometry" and "the Elements of Solid Geometry." The former has already been noticed. The latter is based, as the author tells us in acknowledging his indebtedness in the preface, on the two draft syllabuses of solid geometry of the (English) Association for Improvement of Geometrical Teaching prepared one by C.W. Merrifield, and the other by one signed R.B.H. (R.B. Hayward). Fortunately, there is an English translation of the book by the author himself. So my task is easy; I need only refer to this translation for fuller particulars which any of the readers might wish to know. The author has since been engaged in writing a companion volume to his text-book which is to supply explanations, illustrations, and developments for the convenience of teachers and those who have to study without teacher. Of this book, two parts only have been hitherto published; Part I in 1897 and Part II in 1906.

Geometry as taught in our middle schools is essentially Euclidean in character with accepted modifications such as the preference of Playfair's form to Euclid's twelveth axiom. So long

as the Euclidean geometry continues to be the geometry of practical life, it should be that of schools, notwithstanding that training in pure reasoning is the vital aim of our teaching of geometry. To teachers we do warmly recommend the study of non-euclidean geometries, but not a syllable of even their names should ever be breathed in the presence of schoolboys. It is a sin to rouse the curiousity of schoolboys in matters which cannot possibly be explained to them.

The treatment of Ratio and Proportion for magnitudes in general without regard to the distinction of commensurable or incommensurable, as given in Kikuchi's Geometry, has given rise to much complaint in view of the difficulties of comprehension on the part of pupils. I myself suggested a slight modification of the treatment in a course of summer lectures given in 1889 at the request of the Department of Education. A verbatim report of these lectures was published as manuscript in 1900 during my absence, while I was attending the Second International Congress of Mathematicians at Paris. The seventeenth lecture occupying pp. 387—406 of this report is devoted to this suggestion. Besides myself, there have been several others at work for mitigating the said difficulty. And to this day it is still looked upon as a problem awaiting solution.

Although geometry as taught in our middle schools was framed on the "Association" model, in a certain aspect, it was considerably influenced by continental (European) text-books of geometry. Among German text-books of geometry, none seems to have made more impression than the joint-work of J. Henrici and P. Treutlein, *Lehrbuch der Elementar-Geometrie*, particularly in the direction of making felicitous use of motion in proving geometrical theorems. Treutlein's new book, *Der Geometrische Anschauungsunterricht* promises to be read here with much admiration. Among French text-books of geometry, those of Rouché *et* Camberousse are perhaps the

best known, and some of them were translated into Japanese. The epoch-making works of Méray and Hadamard destined for teachers and the comparatively recent *Géométrie* of Borel's *Cours de Mathématiques* are not entirely unknown. Some of the foreign collections of exercises and problems which seem to be in favourite use here, are those of Julius Petersen, A. Hoffmann, Iwan Alexandroff, J. McDowell, A. Larmor, and others. It may also be added that the text-books of geometry by Franz Hocevar, including the English translation of his solid geometry, are also much appreciated.

Reports on the teaching of geometry prepared by the (English) Mathematical Association Committee, which have been published from time to time, as well as the report on the correlation of mathematical and science teaching prepared by the joint committee of the same Association and the Association of Public School Masters, have served, as is but natural in view of the origin of our way of treatment, as invaluable references.

We now return to the actual teaching of geometry in our own schools. In order to meet the omnipresent complaints that geometry is so hard to teach, there were proposed from time to time very many contrivances, of which a typical example may be found in recommending the use of chalks of various colours on the black-board, and which have the common merit of appearing so ingenious to their originators alone. Strange to say, very similar,——almost the same——sorts of contrivances seem to have been suggested in England in the first half of last century. They are to be found in some antiquated out-of-the-way text-books of geometry. I cannot do better than explain them by referring to a few of such text-books.

I will give the full titles of these text-books as they would give a very good idea of the contents:—

> New Mathematical Demonstrations of Euclid, rendered clear and familiar to the minds of youth, with

no other mathematical instruments than the triangular pieces commonly called the *Chinese Puzzle*. Invented by Mr. W. Williams, High Beech Collegiate School, Essex. London: Printed for the Author; and sold by J. Maynard, Panton Street, Hay-Market, and other booksellers, 1817.

The First Six Books of the Elements of Euclid, in which coloured diagrams and symbols are used instead of letters for the greater ease of learners. By Oliver Byrne, surveyor of Her Majesty's Settlements in the Falkland Islands and author of numerous mathematical works. London, William Pickering, 1847.

It is indeed, to use a colloquial expression, awfully interesting to observe that the same ideas suggest themselves in countries separated by a hemisphere and at intervals apart by nearly a century. No less interesting is it to notice that the germ of the so-called *intuitive-experimental* method or even the method itself is to be found in the books above mentioned, particularly in the one named first, though expressed in words somewhat different from those customarily used nowadays. I should have wished so much to reproduce in full the preface of this book of Williams, if it were not for the fear of going too much astray from the nature of this report.

From what I have just said, is not to be inferred that we slight altogether what is usually known by such various names as experimental geometry, inventive geometry, constructive geometry, etc, etc. Some kinds of schools, such as the technical schools of the lower grade, are the place *par excellence* for this provisory method. Indeed, it would have been expedient, if such were known by some name other than that of *geometry*. Much of the unnecessary and really meaningless controversies would thus be made to disappear altogether. It is much to be regretted that there

are people who stick too much to the name and whose eyes get dizzy in perceiving that the subjects covered by the same name in different kinds of schools may differ widely and must not be confounded.

What we contend, is that the geometry to be taught in the trunk system of our *educational tree* should be *the geometry* which aims at training in logical reasoning, and that we should be prepared to meet half way the difficulties, in overcoming which the training desired finds its very existence. Very often it is argued that all the school work should be presented to boys in the most interesting way. Such is ever more to be desired, if only it can be done. We, adults, too, would like to be amused all the time, but it cannot be helped that there are hours of real earnestness in the course of our life. Moreover, the boys' *interest*, using the word in the pedagogical sense, itself admits of being evolved and cultivated not necessarily by smooth gliding but often by conquering difficulties. An uninteresting dogmatical beginning followed successively by the gradual awakening of the logical faculty hitherto quiescent and the charming consciousness of being able to do what appeared so difficult some time before, is surely worthy of being included within what is called *interest*. Few of the middle school subjects are, we are told, interesting to begin with; so they must be *made* interesting. To the abuse in this *making* are to be traced some of the irrationalities, sophistries, self-contradictions, and fallacies in general, actually found in the discussions, so much in fashion here nowadays, concerning the method of teaching.

As regards the so-called *fusion* of plane and solid geometry, there has been much diversity of opinions, not only in general, but among those who are *pro* as well as among those who are *contra* such a change of established

usage. The argument put forward so ably and with such admirable reserve by Baron Giuseppe Veronese for the change with special reference to his *Elementi*, on the occasion of the Paris Congress of 1900, is much appreciated here. On the whole, however, I think I cannot possibly err when I say that there is a preponderance of views favouring *no* such change for the reasons to be presently explained.

The study of geometry in our middle schools extends over three years, and the duration of three years at the middle school age is quite a long period of time. And during this time, it is most likely that something would occur as regards individual boys. Most of them will have to be away from the school at one time or another on account of illness or some other unavoidable circumstances. Again one morning a boy may feel a headache; another morning some other boy may feel somehow giddy and so sits absent-minded while the class-work is going on in spite of himself. Thus there are innumerable possibilities, under which it cannot be altogether avoided that individual pupils drop behind the progress of the class. From this cause no subject seems to suffer more than geometry, as is only natural. Very often it is hopeless to find out the end of the tangled thread of great length; and it is always advisable to cut such a thread into portions if there be any fear of entangling. Division of big ships into water-tight compartments would be useless, if every thing were always running smoothly. That even such precaution was powerless to prevent the catastrophe of the Titanic, may be compared to the extraordinary sinking of the *average* of an unusual class due to the chance aggregation of abnormal boys utterly irreconcilable to mathematical lessons. The arguments in favour of the fusion would be difficult to refute, even

unassailable, if it could be assumed that the teaching and learning go on without a hitch. But there are rainy days as well as sunny ones. Were it possible, geometry should have been cut into such portions that each of them forms a complete whole, and this should be done even at the risk of overlapping or repetition as was said before. At any rate, the established usage of the division into plane and solid geometry should be kept intact; they should even be separated, if the established usage were otherwise.

Trigonometry

Plane trigonometry alone, and, in fact, its rudiments only, are taught, as the Syllabus given in Chap. IV shows, in our middle schools.

As regards the definition of trigonometry, it may be interesting to observe that, here as elsewhere, we are, not purposely but, so to speak, unconsciously, following the imperishable *Encyclopedia Metropolitana*. In it there is an article on trigonometry by George Biddel Airy, which says that it is the branch of mathematics which treats of the application of arithmetic to geometry. Perhaps, we may have slightly modified this, in as much as we say that trigonometry treats of the application of arithmetic and algebra to geometry. Indeed, it is the connecting link by which we are enabled to combine the practical accuracy of arithmetical calculations with the hypothetical exactness of geometrical constructions through the intermediary of algebraical formulae. If any one wishes to know something of the utilitarian value of algebra or geometry, no far-fetched exertion needs be made, and he is simply referred to trigonometry. The correlation between algebra and geometry is to be found in trigonometry. Such is our attitude toward these three branches of elementary mathematics. Some years ago, trigonometry was struck off by bureaucratic force from our middle school curriculum, on the assumption that it is independent of algebra and geometry, and something higher than these two. Soon afterwards, however, the nature of trigonometry as inseparable from algebra and geometry having been made clear even to those who do not know mathematics, trigonometry was reinstalled in its old befitting place.

A few words of general comments shall close this chapter. Generally speaking, there are teachers in special subjects but not teachers of the class. Very often we hear of the correlation of the subjects taught in middle schools. But what is really wanted is the correlation of the thoughts and actions of the teachers in special subjects. This rather delicate distinction has been mixed up, leading to many perplexities and even abuses. Any amount of correlation on the face of the syllabus is powerless for correlating the actual teachings.

In elementary schools, a teacher is a *class teacher*; he teaches all the subjects; thus there is one teacher to one class. We may say, to use a mathematical phrase, *one-to-one* correspondence. In middle schools, sometimes one teacher teaches more than one subject, but, generally speaking, each teacher teaches only the one which is his special subject. Here we have a *one-to-several* correspondence. The partition of the trunk of our *educational tree* into the *elementary*, the *middle*, and the *higher middle* parts, owes its origin to various complicated circumstances, but I have been always in the habit of thinking that, in discussing this partition, more emphasis should be laid on what I have termed *correspondence* than has hitherto been done. In regard to the middle school, pressure has been brought to bear towards diminishing the number designated by the ambiguous term *several* in speaking of one-to-several correspondence. But this requires the general raising of the standard of teaching capabilities, which is an extremely difficult task under the present system of recruiting middle school teachers.

Besides the *magazine-for-young-folks* evil spoken of before, and the *examination* evil closely connected with it, there is an evil of *congested curriculum*, which affects the average boy equally disastrously. In the middle school, there are now so many subjects, each of which aspires to equal—— even greater——importance, and, so to say, tries to expand

as far as possible——the *imperialism of individual subjects*. And all this is taking place under the practical impossibility of the correlation of the thoughts and ideas of specialist teachers notwithstanding the bureaucratic correlation of subjects on paper. As a consequence, every subject is taught so minutely and conscientiously that the ordinary boy has never the chance of struggling out of the elementary stages of his subjects.

So far as the teaching of mathematics is concerned, we cannot help looking back with something like grief to the days of *Ei-Kwan-Sū*, when one foreign language, the Language (nationalized Chinese), and mathematics were the only principal subjects taught in our secondary schools. Of these three, perhaps, mathematics suffered most by the later addition of so many subjects. Modern education seems to demand the inclusion of all these subjects. Granting that this cannot be helped, there still remains the problem:— whether we cannot check the imperialistic tendencies of the teaching of individual subjects. Will it not be possible to discriminate a little in view of the relative importance, not of the subjects themselves, but of the positions of these subjects in the middle school education? No more is demanded than that the spirit of one-to-one correspondence should reign supreme under the actual, for the present inevitable, system of one-to-several correspondence. And yet this seems to be a delicate matter to handle and no easy task to accomplish.

No account of the teaching of mathematics in middle schools can be even partially complete, without something being said of the *external* competitive entrance examination at the gate of the higher middle school, which makes itself impressively felt all down the line. It is true that the number of the graduates of middle schools who present themselves at this examination is somewhat diminishing. Still the admission into the higher middle school is the

hypothetical goal, toward which the majority of our middle school boys are striving. For those who aspire to prominence among the educated in their after-life, it is the stepping-stone, of which it is by no means an exaggeration to say that it decides their future destiny. The evaluation of the efficiency of a middle school by the number of its graduates admitted into the higher middle school, though it has no real ground, has not lost the force of long accumulated inertia. Of this omniparous examination, we shall, however, speak in the chapter which follows.

CHAPTER VII

THE TEACHING OF MATHEMATICS IN HIGHER MIDDLE SCHOOLS

Higher Middle Schools form a part of the trunk of our *educational tree* lying intermediate between Middle Schools and the different Faculties of the Imperial Universities. The official name is *High School*, but as this has a somewhat diverse specific significance in different countries, I thought it preferable to use the name just given, in order to avoid possible confusion. The higher middle school course extends over three years, in the normal case i.e. according to the regulations on paper, beginning at the eighteenth year of age.

There are at present eight higher middle schools scattered all over the country, namely one in each of the towns of Tokio, Sendai, Kioto, Kanazawa, Kumamoto, Okayama, Kagoshima, and Nagoya, and called respectively the First, the Second, and so on up to the Eighth. Besides, there is a preparatory school forming a part of the Faculty of Agriculture of the East-Northern Imperial University. It is located in Sapporo and is very similar to a higher middle school. All these schools are under the direct control of the Department of Education.

As explained in the introductory chapter, the aim of these schools has passed through many and varied changes. To-day they are preparatory schools for the different faculties of the Imperial Universities.

The qualification for entrance is the maturity of the middle school course. However it is so only in theory; practically boys have to pass a severe competitive examina-

tion, of which we shall speak presently.

There are three sections (called *departments* in the Divisional Report); the first section is for those who wish to enter the Faculty of Law or Literature; the second for those who wish to enter the Faculty of Science, Technology, or Agriculture, or the course of pharmacy in the Faculty of Medicine. [Cf. Chap. I, pp. 6—12.]

The subjects taught in the first section are morals, the Language and classic (or nationlized) Chinese, foreign languages, history, logic, and psychology, outline principles of law, elements of economics, and gymnastics. There are some minor variations in the subjects according to the particular course in the Faculties that they wish to enter. Foreign languages must be two of the three, English, German, and French. Latin may be taught as optional subject to those intending to enter the Faculty of Law. For those intending to enter the course of philosophy in the Faculty of Literature, mathematics and physics are to be substituted in place of logic and psychology.

The subjects in the second section are morals, the Language, foreign languages, mathematics, physics, chemistry, geology and mineralogy, drawing, and gymnastics. There are many variations in the third year, both in the subjects and in the number of instruction hours per week given to different subjects, according to, which particular Course they intend to take up on entering the University. Thus, e.g. those who are going to take up the course of botany, zoology, geology, agriculture, or pharmacy will not have to take any mathematics in the third year, but will have botany and zoology instead, while those who are to become students in a Faculty of Technology will have a great deal of drawing practice, and also begin the elements of theory and practice of surveying, and so on. The foreign languages must be English, and one of the two, German and French.

The subjects in the third section are morals, the Language, foreign languages, Latin, mathematics, physics, chemistry, zoology and botany, and gymnastics. The foreign languages must be German, and one of the two, English and French.

According to the official return for the statistical year 1908-9, the aggregate number of the teaching staff in all the eight higher middle schools was 303, including 192 professors, 13 assistant professors, 73 persons temporarily employed, and 25 foreign teachers. All foreign teachers were instructors of foreign languages; among them, there were 12 Germans, 6 Americans, 5 Englishmen, 1 Frenchman and 1 Swiss. The number of applicants for admission was 9807; 2640 of whom were for the first section, 4005 for the second, and 3162 for the third. Of these, 976 were admitted into the first section, 714 into the second, and 319 into the third; in all 2009. Thus, on an average, one out of every five applicants were admitted. This ratio differs considerably in different sections, being about 1 out of 2.6 in the first section, 5.7 in the second, and 10 in the third. As a matter of course these ratios vary considerably in different schools and also from year to year. The total number of boys in all the eight schools was somewhat less than three times the number of those admitted, which does not differ much from year to year, the actual number in 1908-9 being 5,375. In passing, we may observe that, about this period, the tendency of our best boys preparing for the medical profession reached its maximum. Ever since this tendency has been remaining stationary, and only quite recently showed a symptom of almost imperceptible decline.

Competitive Entrance Examination

The numbers above given would hardly convey an adequate idea of the severity of the competitive examination for admittance into higher middle schools. Among the applicants, there are first of all picked boys from over three hundred middle schools scattered all over the country——the flower of the budding generation. There are those who have failed once, twice, thrice, and so on——I know a case where an applicant failed six times——and who have been preparing all the time during the corresponding number of years and, for all that, are still obstinately persisting in preparing for the examination.

In the case of smooth progress which is not of very frequent occurrence, a boy leaves a middle school with a certificate of maturity at the end of March and has to go through the entrance examination into a higher middle school, which takes place usually in the first part of July. This interval of a little over three months is the hardest time in his whole life, during which he has to make superhuman efforts in cramming. He has to exhaust all the energies which he might have in store, and, very often, this produces an after-effect telling severely both on his bodily health and mental capacity. All through his life, he can never forget this trying period, when he had to work night after night till the small hours of the morning, and, finally succumbing to his utmost exertion, might have slept with his head drooping on his desk.

Not all the graduates of middle schools go to the higher middle school. Some go to the military schools, some to naval schools, some to industrial schools, some to so-called special schools, and so on. But in almost all cases, the number of applicants for admission is far in excess of the number of those who could be admitted. So in every such

case, there is always a competitive entrance examination. The keenness of the competition varies as a matter of course in different entrance examinations, and this is no less due to the character of the school and the corresponding, so to speak, material of the applicants than to the ratio of the number of applicants to that of those admitted. The competitive examination at the gate of the higher middle school is to be looked upon as a typical example. So what will be said presently, applies to all other such examinations with the only difference in the degree of keenness. Hitherto, the competitive entrance examination for the higher middle schools has always been considered to be the hardest of all. But I was told that, in recent years, the competition is almost as keen in some naval schools as in higher middle schools.

In the statistical year to which we have been referring, the number of the graduates of middle schools was approximately fifteen thousand, while that of the applicants for admission into higher middle schools was about ten thousand. This would give an appearance of two out of every three graduates of middle schools applying for admission into higher middle schools, which is erroneous. As was said just now, this number ten thousand of the applicants owes its magnitude to the accumulation year after year of the graduates of former years, who have been thus far unable to attain their long cherished end. In fact, only about eight hundred of the year's graduates were admitted in this year.

It is only natural that such a competition awaiting the boy's leaving middle schools should exercise a powerful influence upon the teaching in middle schools in general, and particularly upon the teaching of mathematics as will be forthwith explained. Again, wherever there is an interest of life-or-death, evils are sure to creep in, often under the mask of innocent assistance. As they too

throw back their shadow on middle school teaching, we will notice some of the most abominable of these evils.

First of all, there is the evil which I cannot better describe than by naming it the *evil of mathematical dictionaries*. There are dictionaries of arithmetic, algebra, geometry, and, trigonometry, and of all of them taken together. They are nothing more than collections of problems with solutions catalogued and arranged in such a way that it is worthy of the name of *dictionary*. Every one of them is not a key to any one particular text-book, but is something like a key to all the text-books on the subject with some more problems than are found in them. In these dictionaries, problems are so ingeniously arranged that, given a problem at random, it can at once be found as easily as, to use a vernacular expression, searching for things in a bag. The inventiveness shown in the mischievous work of compiling such a dictionary is indeed a marvel. It is a pity that the same inventiveness has not manifested itself in less harmful undertakings, if not useful. The evil arising from the existence of such dictionaries is obvious. Boys do not think out problems but immediately search for ready-made solutions. It would be cruel to suggest to teachers of mathematics that they should give boys new problems. Just think of teachers in the English language forbidden to use words to be found in the new Concise Oxford Dictionary! The climax of the evil is to be found in noticing boys who might have the bitter experience of repeated failures, doing their best by cramming all the solutions contained in a mathematical dictionary.

Another evil, no less harmful, is the coaching for examination's sake, which has been ever on the path of doing mischief and attained the present state of perfection. Particularly in mathematics, this evil is most keenly felt. In larger cities especially, there are great many such professional coaches eagerly searching for applicants. Some

of them who have obtained a high reputation (?) among boys, do marvels. In preparing boys for entrance into a particular school, they make a careful study of problems set during the last five years or so, and also of the personal inclinations or any mental peculiarities of examiners as well as whoever might have a hand in the selection of problems. How they sometimes succeed in predicting the inner thought of others, is really wonderful. Human ingenuity was supposed to be unfathomable, but it seems as if the bottom has been touched in this case. The prediction seems to be comparatively easy in the case of geometry as compared with any other subjects. This may be at least partly due to the limited choice of problems in geometry. Some years ago, there was an accidental coincidence of the same two problems and only these two in geometry set to applicants for admission into two altogether different kinds of schools. This fact seems to indicate that teachers of mathematics in schools higher than the middle school, whose daily work lies beyond the scope of elementary geometry, on meeting with the untimely need of choosing a few problems in geometry, have a comparatively limited latitude of choice. Thus for an expert in this line of guess-work, it does not require a Herculean effort to find out, so to speak, approximately, if not just hitting the Tell's apple, the problems which will be set in any particular year. He selects probably from ten to twenty such problems in geometry, which, together with their solutions, his pupil is required to commit to memory. On the critical day, the pupil feeling languid and perhaps not a little nervous, enters the room where the entrance examination which may prove fatal is to take place. Printed questions are distributed among the occupants of the room. As soon as he sees the paper, there——*Gott sei Dank!*——there are just two or three questions out of the ten which he had with so much exertion committed to memory. All his old vigour is

now restored to him. Right on, he writes down the solutions without thinking and as rapidly as he can, all the time admiring the wisdom of his coach, who, at this moment, appears to him like something divine. Long before the prescribed hour is out, he leaves his place leaving his comrades still lingering in the doomed room. Those experts who were lucky enough to hit upon the largest number of questions, now become the object of everyday talk among boys and are hailed as great coaches.

To those who have had no such experience, it would seem rather strange that the competitive entrance examination should give rise to difficulties on the part of examinees as well as on the part of examiners. The evil described above is well-known to the examiners. So they try to counteract it, but, in so doing, they are struggling against their superiors in this line of work. Questions are made stiffer, or more crooked, so to speak. But then, this seems somehow to give more facilities in guess-work on the part of experts. We cannot help admiring the truth of the saying: 'Those who try to evade anything are, as a rule, cleverer than those who try to stop such evasion.

It has been often argued that boys' attainments in mathematical subjects can be graded more definitely than in any other subjects. I, for one, do not believe in this; but I shall let pass in silence this dogma which is accepted by so many people. Because of the facility of the grading of the results, importance was given to mathematical subjects in competitive examinations. To the same cause it is due that, for some time past, there has been a tendency of making the mathematical questions set at such an examination a trifle more difficult. However, a slight bureaucratic pressure having been applied to thwart this tendency, some years back, they have been made far easier than they used to be. Those who hate all examinations indiscriminately, feel satisfied as long as they see

examination questions made easier and easier. To use a mathematical idea, they wish, by continuing the process of *easifying* indefinitely, to reach the limit of no examination. But this involves self-contradiction, at least, as concerns competitive examinations. Where competition is unavoidable, equitable grading of competitors' attainments is necessary; by making examination questions too easy, the very purpose of examining for the sake of fair grading will be defeated, and it will lead to the consequence worse than deciding by lot. Glancing back through a little longer period, say a decade and a half, it would hardly escape the notice of an acute observer that, in the degree of hardness of examination questions, there has been a series of fluctuations in one way or the other. All the time, however, the watchful coaches have never lost their grip of the thread's end, adapting themselves so well to the attitude of examiners and even forestalling changes of their attitude.

Again here is what may perhaps be called a peculiar phenomena in absence of any thinkable name, which, so far as I am aware of, is to be found nowhere else in the world. I will not say a positive evil, but I think I can say so much that it would be better for all of us if there were no such phenomena. We often hear of mock battles and sham fights; but few people may have heard of mock or sham entrance examinations, not to say a course of such examinations, that is to say, a course in which there is nothing but holding such examinations from day to day. What may perhaps be called the "centralization of educational opportunities" reigns in our country. In this respect, Tokio in Japan is very much similar to Paris in France. Now in the very midst of our educational center, there are schools which are fortunately few in number, where such courses are to be found. In one of them sham entrance examinations in various middle school subjects are held on every Saturday and Sunday. Every week make-believe

examinees have to pay fees. Of the particular subjects in which mock examinations are to take place in a particular week, previous warning is given and the interval from Monday to Friday is intended for preparation. Feigned examination papers are *said* to be carefully examined, defects pointed out, remarks added, and model solutions supplied, and all put into a sealed envelope is returned to each examinee. Examinees are *said* to be thus disciplined in the way of writing answers, in being accustomed to examinations so that they would no longer feel nervous in the real examination room, and in gaining that self-reliance which dispenses with longing for haphazard chance. Now all this sounds fine, and indeed there is nothing to be said against what stands on a printed circular. Such circulars announcing the existence of such a course are sent directly to boys just in the nick of time, that is, on the eve of their graduating from a middle school, enticing them to come to the schools for paying admission fees, which must not, however, be sham in spite of the examinations. As before said, luckily, we have but a very few such schools, and the mischief hitherto done is insignificant compared to the magnitude of other evils. This is merely to show, to what extremity cramming, cliquish instruction in *pass* secrets, and, in short, every means of outwitting the examiners, have gone, and all for defeating the very purpose of competitive entrance examinations. The evils have taken their own course out of all nick. Now, however deplorable the direct effect of these evils are, it would appear almost insignificant compared to the gigantic indirect influence which such evils seem to exercise upon the teaching, in particular the teaching of mathematics, in middle schools.

How are the evils of competitive entrance examinations to be done away with? Since they come from there being more applicants than can be admitted, some would suggest: Increase the number of higher middle schools. Plausible

as this solution of the problem may seem, it can only fit the exigencies of the hour. The real remedy is to be sought in a quarter least surmised. But the nature of this report forbids me to go any deeper into this question. Perchance I may have already gone too far astray from the path I ought to have trodden.

The Curriculum in Mathematics

The teaching of mathematics in higher middle schools has chiefly to do with students of the second section. In the third section, mathematics is taught, but not so much as in the second section. In the first section, it is taught just a little to some students who form only a small fraction of the whole. So in the following we shall follow the order: the Second Section, the Third, and finally the First.

The Second Section

The number of instruction hours per week assigned to mathematics is 5 in the first year, and 4 in the second and the third.

The subjects of study include Trigonometry, Algebra, Analytical geometry, and Differential and Integral calculus.

The number of hours in one school year is about 160 in the first year, and 128 in the second and the third, making a total of about 416 hours in all the three years. Out of these are allotted, to trigonometry 63 hours in the first year, to algebra 128 hours in the first and the second year, to analytical geometry 97 hours in the first and the second year, to calculus 128 hours in the third year.

In trigonometry, Todhunter's "Plane Trigonometry for the use of colleges and schools" is used as the text-book.

This would give a tolerably fair idea of what is done in this subject to all those who know of this well-known English text-book. The characteristic beauty of whatever Todhunter wrote, particularly his painstaking exactness which shows itself in the proverbial absence of even a single typographical error in all his voluminous writing, is deeply admired here. We have no idea of changing this book, however old, for any of the text-books of trigonometry, of which there are perhaps too many nowadays. Of the knowledge of trigonometry which students ought to have got in the middle school, but slight use is made, and the subject is taught again almost anew.

In algebra, the following text-books are used:

R. Fujisawa——A Sequel to Elementary Algebra.

I. Todhunter——Algebra for the use of colleges and schools.

C. Smith——A Treatise on Algebra.

Fujisawa's Sequel which is written in Japanese is used in connection with reviewing what is taught in the middle school. Formerly Todhunter's "A Treatise on the Theory of Equations" and also Burnside and Panton's "Theory of Equations" were used. The word *algebra* is here used in the sense as it is usually understood on the European continent, and covers algebra and the theory of equations according to the English usage in the days of Todhunter. We have now no fixed text-books of the theory of equations; but I believe the two books just mentioned are still used as valuable books of reference in at least some of the higher middle schools. A rough idea of what is taught under the name of algebra may be got from the following list of headings:

Equation of the First Degree. Inequalities. Equation of the Second Degree. Rational Integral Functions. Permutation and Combination. Binomial Theorem. Theory of Determinants. Theory of Probability. Theory of Numbers. Simple Continued Fractions and

Indeterminate Equations. Convergency and Divergency of Infinite Series. Complex Number. Rational Integral Functions. Properties of Algebraic Equations. Symmetric Functions of Roots. Transformation. Cubic and Biquadratic Equations. Properties of Derived Functions. Separation of Roots. Solution of Numerical Equations. Methods of Elimination.

For details see Divisional Reports, Art. III, pp. 8—14.

In analytical geometry, the following text-books are used.

D. Kikuchi——Analytical Geometry, Plane and Solid. 2nd ed. Tokio, 1912.

R. Miwada——Solid Analytical Geometry.

G. Hale Puckle——An Elementary Treatise on Conic Sections and Algebraic Geometry.

W. Steadman Aldis——An Elementary Treatise on Solid Geometry.

The books of Puckle and Aldis are, I believe, very well known even now in England. An approximate idea of what is here mentioned as *analytical geometry* may be given by saying that the major part of the contents of Puckle's plane geometry and about the first half of the contents of Aldis' solid geometry are taught under this name. This refers to the subject-matter only. In regard to the method of treating or the spirit of treatment, it seems as if slow but steady changes are going on in the direction of analysing the subject-matter into the metrical and projective elements and treating the projective properties from a more general point of view. It may perhaps be otherwise described as the tendency which is to be inferred from the change of *trilinear co-ordinates* or *abridged notation* to *homogeneous co-ordinates*. This is only natural, since such books as Cremona's Projective Geometry and Reye's Geometry of Position, though not used as text-books, have long been in favourite use as books of reference among higher middle school teachers. I cannot help it that the name just mentioned

rouses always in my mind the recollection of the pleasant days some thirty years ago when I was a student at the Strassburg University. I wish to be allowed to add that I feel joy whenever the idea crosses my mind that I am his old but unworthy pupil and that I might have been instrumental, however slightly, in transmitting to many of the higher middle school teachers in mathematics his singularly elegant method of expounding projective geometry.

In differential and integral calculus, the text-books used are those of Todhunter and Benjamin Williamson. The chief books of reference in current use, not necessarily confined to higher middle schools, seem to be the following:

L. Kiepert——Grundriss der Differential-und Integralrechnung.

E. Czuber——Differential-und Integralrechnung.

J.-A. Serret——Cours de Calcul différentiel et intégral. [The French original and the German translation.]

P. Appel——Élément d'Analyse mathématique.

J. W. Mellor——Higher Mathematics for students of chemistry and physics.

W. E. Byerly——Elements of the Differential Calculus and Elements of Integral Calculus.

H. Lamb——An Elementary Course of Infinitesimal Calculus.

A. G. Greenhill——Differential and Integral Calculus.

E. Sakai——Differential and Integral Calculus [in Japanese].

As was said before, [see p. 13] the last year i.e. the third in higher middle schools corresponds roughly to the first year in some of the German universities and polytechnic schools, or perhaps to the sophomore year in some of the American colleges and universities. Now as many as about one hundred thirty hours in this last year given to the subject ought to impart to students a tolerably sound knowledge of the *ordinary* calculus. Some years since, I have been

requested by the Department of Education to inspect the higher middle schools. I have already inspected four of them, and I may have to inspect the remaining four or five when the present work will be over. Speaking from my own personal experience thus far gained, it seems, there still remains much room for wishing that the teaching of calculus should become more effective.

Why the teaching of calculus is not so effective as could be reasonably desired, is, it seems to me, due partly at least to the absence of suitable text-books. It needs hardly be said that text-books are not used in higher middle schools in the same way as in schools of the lower grade. A text-book is followed, so to say, broadly and not too closely. Instruction is very often given by means of lectures following in outline the text-book in use. Even problems for exercises are not necessarily taken from it. And still a text-book which just suits our higher middle school is very much desired. That we are still using Todhunter's and Williamson's text-books, which, as may be noticed in passing, dates from the days when I myself was taught calculus, is not because we are contented with them, but is due partly to the force of inertia and mainly to the cause which will in some measure be explained hereafter. So the actual instruction deviates considerably from that which might be inferred from the use of the above text-books.

For the beginning, it was evident that no text-books for general use would answer our purpose. To avoid misunderstanding, let it be clearly said that distinction is to be made between a text-book on calculus for general use which may serve as a very useful book of reference and for which any good foreign text-books may be used, and a class text-book which should adapt itself to the exact want of our higher middle schools. And I am going to speak of such class text-books alone in what follows.

I have been often urged to write a text-book of calculus,

and indeed there seems to be even a censure in some quarter that I have not done so as yet notwithstanding my long cherished desire to do so and in spite of my having so often thought of it for many years past. I hope some one, perhaps some one of the higher middle school teachers in mathematics who have had a long experience, would do this work. So, for his benefit, let me narrate some of the difficulties which presented themselves in the course of my contemplation in draughting the main features of the desired compilation.

First of all, there is the difficulty of the language in which such a book should be written. For many years to come, there is the practical necessity that one for the use of the second section should be written in English. This difficulty may however be overcome somehow. Then follows the real difficulty of compilation. Originality in such a work of compilation is altogether out of the question; but originality in the mode of treating or presenting the subject, especially as regards the adaptability to the use of a particular class, is much to be desired and is not out of the reach of one who has patience in turning his long experience into account.

We may distinguish three characteristics in the spirit of compilation, namely, straightforwardness, ingenuity, and stuffing. Straightforwardness consists in following persistently one line of a certain preconceived principle without looking either to the left or to the right. Such may be found in Todhunter's Differential Calculus, where the method of limits or differential coefficients is used consistently throughout the book to the almost total exclusion of the method of differentials. We have long given up following this book in excluding differentials. Nevertheless, the spirit of straightforwardness, but not the actual treatment, just mentioned, has taken such a strong hold of us, that we still see the name of this book in the list of class text-books.

Stuffing is to be found in accumulating as much material for the text and problems for exercise as is compatible with the number of pages which might have been premeditated, looking constantly to right and left in the anxiety of having missed something which ought to be included——an essential requisite in the make-up of those who compile books of encyclopædic purpose. If I may be allowed to say what has been lingering in my mind for such a long time, there seems to be something of this stuffing tendency in Williamson's Calculus. On this very account, however, it is invaluable for some kind of readers, although it is not suitable for class use in our middle schools. Thus, both Todhunter's and Williamson's Calculus, though still standing on the list, are not much made use of in actual class-work except in so far as problems for exercise are very often taken from them.

Ingenuity of treatment would hardly require any elucidation. Only it must not go to an extreme. Ingenuity in a text-book for class use, if it is to be used efficiently, presupposes ingenuity in comprehension on the part of the majority of students in a class——which is of rare occurrence in real actuality. Delicate instruments must be delicately handled; ingenious contrivances are liable to get out of order; an amateur sees nothing in a higher power microscope or telescope. However, some ingenuity, to which a class may become equal, but not too much, is, of course, greatly desired. Now, straightforwardness in the first place, then a certain degree of stuffing and then some ingenuity are the requisites of the future text-book for the use of the third year students in the second section, the compilation of which awaits the patient and disinterested labour of some pioneers in this line of work.

Todhunter's accuracy in the absolute absence of typographical errors may not be attainable, particularly in view of the difficulty which may be imagined by thinking of the printing of our own ideographic characters

done in America or Europe. Still utmost care should be taken in freeing the future text-book just spoken of from misprints. And the absence of typographical error, however desirable it is for its own sake, is to be looked upon as typifying manifold care which should be bestowed upon the work of compilation. Problems must not be put in *ex tempore*. The presence of a few irrelevant problems, even if they may not involve positive error, would destroy the confidence of students in the book. Psychology has been so often applied to educational matters that we even hear of a misconceived psychology in this connection. It seems rather strange that the psychological phenomena which may appear in the relation of a class text-book *versus* the class, has been sometimes not sufficiently taken into account. A class text-book should command the respect and confidence of the class for the smooth working of instruction, just as the same must hold good with respect to the teacher. Such things as the discovery of over-confidence on the part of students should never occur. Now having written so far, this very moment I cannot help thinking back, with something like profound admiration, to so many text-books written by Isaac Todhunter, though some of them are now antiquated.

We will now speak of the teaching of mathematics in general in higher middle schools. In the drift of giving instruction, we may distinguish two kinds of tendencies. One is to drill students in masses and produce things equal to sample; the other is to let individual students develop according to their inborn ability, gifted ones progressing as fast and as far as they can, leaving their less gifted comrades behind. The education given in our higher middle schools (also to a certain extent in our universities) is of the former type, and is necessarily so in view of the fact that

the aim of these schools is to give preparatory instruction for entering one of the Imperial universities. Without even questioning its merits or demerits, we have to rest contented at least for some time to come, thinking that this is inevitable under the present organization of our educational system. So we had, in the above, to think of class-work, to speak of class text-books, and so on. The requisite of the future text-book of calculus, of which we had to speak so much, may perhaps be covered by saying that it must be something like a military drill-book deprived of its monotony and dryness and embellished with a little fat and flesh.

As was said before, the instruction in mathematics is, generally speaking, given by lectures accompanied by exercises in solving problems. Now the lecture system is uneducational on account of the taking of notes. Ideally, the lecture ought to be listened to without any note-taking, except possibly for a few headings and should be reproduced afterwards by the students themselves in their own language and put down in their notes. Where text-books are used conjointly with lectures, they should be made use of in dispensing with the greater part of note-writing, and the lectures themselves must be given keeping in view the accompaniment of such text-books. What has been just said of lectures does not, as a matter of course, apply to the exceptional cases, where a great mathematician makes use of his lectures as the only channel through which his discoveries are made known to the world.

The Third Section

The third section comprises all the students who are going to enter the medical faculties of the Imperial universities. The dominating foreign language taught in this section is German. The number of hours allotted to mathematics in this section is considerably less than the corresponding

number in the second section. Namely, 3 hours a week in the first year and 2 hours a week in the second year, in all about 160 hours, are given to this subject. Of these, about 40 hours in the first year are assigned to trigonometry and algebra, about 56 hours in the first year to analytical geometry, and about 64 hours to differential and integral calculus. Text-books at present in use are Kambly's Elementar-Mathematik I (Arithmetik und Algebra) und III (ebene und sphärische Trigonometrie), Smith's Algebra, Todhunter's Trigonometry, and the Höhere Analysis (Differential- und Integralrechnung) I und II of "Sammlung Göschen."

How to utilize about 160 hours allotted to mathematics in the first and second year of this section in the most efficient manner beneficial to the aggregate of the future students of medicine, is a problem which still awaits a satisfactory solution. The details of teaching given on pp. 32—39 of the Divisional Reports, Art. III, are by no means to be looked upon as definitive. In teaching calculus, since some years, the *Höhere Analysis* of the Göschen Sammlung has been followed by way of experiment, but it has not proved to be entirely satisfactory. The proper distribution of hours among different branches of mathematics is also a question of much importance. Some may suggest the mixing up, or non-distinction of, different branches. But then here we are not allowed free hand in dealing with the question. It might happen that some of the subjects are to be taught to a mixed class composed of elements belonging to different sections in order to economize the instruction capacity of the teaching staff. Several years ago, I suggested to some of the middle school teachers in mathematics Jules Tannery's "*Notions de Mathématiques*" as a useful book of reference.

The First Section

In this section, mathematics are taught only to a select few who are going to enter the course of Philosophy at a faculty of Literature in an Imperial university. Two hours a week in the first year, in all about 62 hours are given to teaching just the rudiments of analytical geometry and infinitesimal calculus. No text-book is used except in one out of the eight higher middle schools, where Junker's *Höhere Analysis* of the *Göschen Sammlung* already mentioned is partly used.

It was in connection with the teaching of elementary ——it may be more appropriate to say it rudimentary—— calculus, that I was asked a few years ago to suggest a suitable text-book. I made a collection of such books as "Calculus Made Easy" by an anonymous writer who calls himself F. R. S., "The ABC of the Differential Calculus," A Preliminary Course in Calculus" and the like. It was on this occasion that my list of books on elementary calculus spoken of on page 58 was printed for circulation among higher middle school teachers in mathematics. In passing, we may notice that, outside the schools belonging to the main trunk of our *educational tree*, Edser's Elements of Differential and Integral Calculus is used more or less.

The book just mentioned by a certain F. R. S. is a curious one. To begin with, to its title are appended the words "being a very-simplest introduction to those beautiful methods of reckoning which are generally called by the terrifying names of the differential calculus and the integral

calculus." Again on the page following the title page, where perhaps a solemn dedication to some illustrious personage might find its proper place, one meets to his astonishment the following remarkable passage:

What one fool can do, another can.

(*Ancient Simian Proverb*)

This reminds me of an old anecdote which I heard when I was quite a young boy, that a stroller, on being queried what F. R. S. means, instantly answered "it may mean fellow remarkably stupid."

I should not have dared to quote what I have just quoted, if it were not for the fact that it so fitly illustrates the spirit, prevalent nowadays here in Japan as elsewhere, of never ending demands in everything to be practical, more practical, still more practical, more and more practical, ..., and any number of *more* practical. Now the tendency to make things practical is very much to be desired in many cases; but often in the very attempt to make things more practical, they are, as in the present instance, made only apparently more practical but in reality more unpractical.

I believe it is due at least partly to the cause just mentioned that the last decade or so has seen the publication of so many "Easy Calculus." Here in Japan too, there have appeared during this interval quite a number of such books. In speaking of these books, exception is to be made of such an excellent booklet as "A Brief Introduction to the Infinitesimal Calculus" designed especially to aid in reading mathematical economics and statistics by Irving Fisher, Professor of Political Economy in Yale University. In passing, it may be noted that the Japanese translation which is mentioned in the preface to the third edition of the original as being in preparation, and which was to have been done by the author's pupil Mr. N. Ishiike of the Kioto Chamber of Commerce, has not thus far appeared. If ever

there be talk of teaching the rudiments of calculus to the students in the first section, who are going to study economics on entering a university, I would not hesitate even an instant in recommending this book of Professor Fisher for class use.

The existence of a large number of Easy Calculus mentioned above seems to testify the presence of the real need of the so-called "just a bit of calculus" in many instances. In fact, we are already confronted by the difficult problem of how to give a general idea of calculus to a group of the future students of philosophy. And surely it lies within the scope of the present enquiry to find out how the rudiments of calculus may be taught so as to meet various demands of such knowledge in very many directions not necessarily confined to the limit of school teaching.

First of all, the question is whether just a bit of calculus can be so taught so as to be of real service to those who learn it. If it be simply for the sake of the *name* that something bearing the dignified title of calculus has been either taught or learned, there is nothing to be said. If it be merely for the purpose of habituating learners to the sight of the symbols of differential and integral calculus so that they shall no longer be terrified by meeting such symbols elsewhere, there remains little to be commented upon. Now when something has been taught to somebody, under ordinary circumstances, it is required that he should be able to apply the knowledge thus gained to some end. But here such is almost out of the question. For any ordinary people calculus is surely a difficult subject to learn; but just a bit of calculus is far more difficult than calculus itself and infinitely more difficult if it were to be turned into account in the sense that the learner is going to apply it in future work of his own. Just think of the present instance of one who learned a bit of calculus in a higher

middle school applying afterwards his knowledge to the profound study of philosophy.

The question converges to this: what should be taught under the heading of just a bit of calculus? Either some portion of the beginning of some ordinary text-book of elementary calculus may be given, or a bird's-eye view of the more important of the whole of such a book may be exposed. Either the almost mechanical part of the processes of differentiation and integration may be taught or the idea which lies at the root of such processes may be expounded without the use of formal symbols as far as is possible. Even the illusory ingenuity shown in that book, wherein fools are so much praised, by saying such things as "the symbol of differentiation merely means *a little bit of*," does not help to emerge out of the ordinary treatment of calculus. If ever one comes across this curious tractate, he is advised to notice the gulf between its tone and what it really contains. He is further recommended to peruse an interesting epilogue and apologue to be found at the end of the booklet, wherein the author dreams of a would-be antagonist in the person of some professional mathematician and makes a furious attack upon his sham opponent. On the other hand, for the exposition of the fundamental ideas without the formal use of symbols, however difficult it may appear, we might find a source of invaluable materials in the calculus of the days of Euler and Bernoulli. It seems that the inclusion or the desire of including something of the history of mathematics in school teaching has taken on too much of the flavour of the mathematical-recreation tendencies. It may and ought to serve the purpose of really improving the teaching of mathematics.

Much will depend upon whether a little bit of calculus is followed by such a subject like mechanics as in the case of technical schools of the so-called middle grade or whether it stands isolated as in the case of the future students of philosophy. Such a thing as one and the same bit of calculus

for all requirements is of course out of the question. In fact there seem to be many ways of approaching as well as of solving this difficult problem of "a little bit of calculus." This same epithet should signify contents considerably different from one another according to individual cases. Thus there is a real reason for our having a great variety in easy calculus. Only most of the books which are actually in existence do not come up to this requirement. They show too much similarity in showing traces of being extracts from some ordinary calculus. As far as we are concerned, we have to find out suitable easy calculus for the future students of philosophy in the first section as well as for the future students of medicine in the third section, all the time keeping in view the obstructive principle of "producing things equal to a sample," which is inevitable under the present educational organization.

[As was remarked before, we have not touched upon the questionable "Revision of the Educational Organization" which has most to do with higher middle schools. I have every reason to believe that almost all who have or had anything to do with education in higher middle schools, looked upon this revision as an experiment inconsistent with all the dictates of prudence and opposed to all the teachings of experience. On the day when the die was cast accompanied by the sole consolation that some time is to elapse before the revision will be actually carried into effect, I can well imagine how miserable they might have felt in thinking that in the near future they should have to surrender their profound convictions. A proverb says: "ways may be found when the end is reached." We hope, the ways of coming out of the death struggle against the firm conviction of so many of our educationists, may be found, before the end will be reached, in the timely postponement, even if not indefinite, of the carrying into effect of the said revision.]

CHAPTER VIII

THE TEACHINGS OF MATHEMATICS IN THE FACULTIES OF THE IMPERIAL UNIVERSITIES

I can but skim over such a wide subject as the teaching of mathematics in the different faculties of the Imperial universities. In fact, the present chapter may be looked upon as a mere reproduction of the Divisional Report prepared by Professor Yoshiye and Dr. Nakagawa, of which the Japanese original has been already published as a separate pamphlet and the English translation was, for the sake of convenience, to be incorporated in this report.

The first article of the Imperial Ordinance for the founding of Imperial universities reads as follows: "Imperial Universities shall have for their objects the teaching of such arts and sciences as are required for the purpose of the State, and the prosecution of original research in such arts and sciences."

The second article of the same Imperial Ordinance runs as follows: "Each Imperial University shall consist of a University Hall and Faculties; the University Hall being established for the purpose of original research, and the Faculties for instruction, theoretical and practical."

There are four Imperial universities at present, namely the Tokio, the Kioto, the Tōhoku (East-Northern), and the Kiushiu Imperial University. The Tokio Imperial University is the oldest institution of the kind and has passed through various stages of development in arriving at its present condition; the Kioto Imperial University comes next, having

been founded in 1897; and the two last mentioned have only been recently established and are in the course of being completed.

The Tokio University consists of a University Hall and six Faculties, namely of Law, Medicine, Technology, Literature, Science, and Agriculture. [Cf. pp. 7-8]. The Kioto University consists of a University Hall and four Faculties, namely of Law, Medicine, Literature, and Science-and-Technology The Tōhoku University consists at present of the Faculty of Science domiciled in Sendai and of the Faculty of Agriculture located at Sapporo. The Faculty of Agriculture of the Tōhoku Imperial University is the name which was given to an old institution, founded as early as 1872 and till 1907 known by the name of Sapporo Agricultural College, on its being incorporated into the then newly established university. The Kiushiu University in Fukuoka contains at present the two Faculties of Medicine and Technology. Each faculty contains a certain number of different and, generally speaking, distinct Courses, ranging from two to ten. The length of a Course is either three or four years. The academic year begins on the 11th of September and ends on the 10th of July next year, with the summer vacation of two months intervening. The condition for admittance is the completion of the preparatory course in a higher middle school, corresponding to the University Course into which admittance is desired, or equivalent qualification usually tested at the higher middle schools. In case there are more applicants than can be admitted, competitive entrance examination takes place.

The reader who may wish for further information is referred to the Calendars of the respective universities. The Universities of Tokio and Kioto publish their Calendars in Japanese annually and those in English* biennially. The

*On sale by Z. P. Maruya & Co. Tōri Sanchōme, Nihonbashi-ku, Tokio.

Calendars (in Japanese), published for the first time, of the Kiushiu University and of the Faculty of Science of the Tōhoku University, reached the writer while this report was being printed.

Tokio Imperial University.

The academic year is divided into three terms; the first extending from the 11th of September to near the end of the year and followed by the new year's vacation lasting about two weeks, the second from some day in the second week of January to near the end of March and followed by the spring vacation of a little over one week, and the third from some day in the second week of April to the beginning of the summer vacation in July.

Besides the graduates of a higher middle school, graduates of the Nobles' School may under special conditions be admitted. The number of such applicants is, however, very small compared to that of those who come from higher middle schools. Postgraduates of the Artillery and Engineering College of the Army Department may be specially admitted to the Faculties of Technology and Science.

Mathematics is taught chiefly in the Faculty of Science, only to a very limited extent in the Faculty of Technology, and just a little in the Course of Forestry in the Faculty of Agriculture. We will begin with the Faculty of Science.

Faculty of Science

The following nine Courses, each of which extends over three years, have been established in the Faculty:— Mathematics, Astronomy, Theoretical Physics, Experimental Physics, Chemistry, Zoology, Botany, Geology, and

Mineralogy. Mathematics is taught in five of these Courses.

Below we give the curriculum of the Courses, in which mathematics is taught. The figures give the number of lecture hours per week. Where the number of times such as *once* or *twice* is given, there is usually no definite limit of time.

Course of Mathematics.

First Year

	1st Term	2nd Term	3rd Term
Differential and Integral Calculus	4	4	4
Analytical Solid Geometry	4	—	—
Projective Geometry	—	2	2
Select Chapters in Elementary Mathematics	—	3	3
Astronomy and Least Squares	3	3	3
General Physics	3	3	3
Mathematical Exercises			

Second Year

General Theory of Functions and Theory of Elliptic Functions	3	3	3
Differential and Integral Calculus and Differential Equations	2	2	2
Theory of Numbers and Algebra	4	4	4
Higher Geometry	3	3	3
General Dynamics	3	3	3
Exercises in Dynamics	Once	Once	Once
Laboratory Work in Physics	Twice	Twice	Twice
Mathematical Exercises			

Third Year

General Theory of Functions and Theory of Elliptic Functions	2	2	2
Theory of Differential Equations	3	3	3
Theory of Numbers and Algebra	4	2	2

	1st Term	2nd Term	3rd Term
Higher Geometry	2	2	2
Mathematical Seminary	Once	Once	Once
Mathematical Exercises			

Course of Astronomy

[The mathematical subjects only are here given.]

First Year

	1st Term	2nd Term	3rd Term
Differential and Integral Calculus	4	4	4
Analytical Solid Geometry	4	—.	—
Mathematical Exercises	Thrice	Thrice	Thrice

Second Year

Differential and Integral Calculus	2	2	—
Applied Differential Equations (optional)	2	2	3
General Dynamics	3	3	3
Exercises in Dynamics	Once	Once	Once

Third Year

General Theory of Functions and Theory of Elliptic Functions	3	3	3
Celestial Mechanics	3	3	3

Course of Theoretical Physics

[The mathematical subjects only are here given.]

First Year

	1st Term	2nd Term	3rd Term
Differential and Integral Calculus	4	4	4
Analytical Solid Geometry	4	—	—
Mathematical Exercises	Thrice	Thrice	Thrice

Second Year

Differential and Integral Calculus and

	1st Term	2nd Term	3rd Term
Differential Equations	2	2	—
General Theory of Functions and Theory of Elliptic Functions	3	3	3
General Dynamics	3	3	3
Applied Differential Equations	2	2	3
Exercises in Dynamics	Once	Once	Once
Exercises in Applied Differential Equations	Once	Once	Once
Third Year			
General Theory of Functions and Theory of Elliptic Functions (optional)	2	2	2

Course of Experimental Physics

[The mathematical subjects only are here given.]

First Year

	1st Term	2nd Term	3rd Term
Differential and Integral Calculus	4	4	4
Analytical Solid Geometry	4	—	—
Mathematical Exercises	Thrice	Thrice	Thrice
Second Year			
Applied Differential Equations	2	2	2
General Dynamics	3	3	3
Exercises in Dynamics	Once	Once	Once

Course of Chemistry

First Year

	1st Term	2nd Term	3rd Term
Elements of Calculus and Differential Equations	3	3	3

In some special subjects, lectures are given to the students of one particular Course; but, in general, lectures are given to mixed groups of students consisting of those in different Courses. For example, lectures on "Differential and Integral Calculus" in the above schedule are given to the first year students in the Courses of Mathematics, Astronomy, Theoretical Physics and Experimental Physics; lectures on the first part of the "General Theory of Functions and Theory of Elliptic Functions" are given to the second year students in Mathematics and Theoretical Physics and the third year students in Astronomy. Below are given the subject-matters of each subject given in the above schedule. They vary somewhat from year to year; so what is here given is to be looked upon as conveying only a rough idea.

Differential and Integral Calculus [*First year* Mathematics, Astronomy, Theoretical Physics, and Experimental Physics] ——limits, continuity, differentiation, change of variables. Infinite series and products. Expansion of functions in series. Evaluation of indeterminate forms. Theory of maxima and minima. Application to the theory of plane curves. Definite integrals. Method of integration. Quadrature and rectification of curves. Double integrals. Cubature and quadrature of surfaces. Multiple integrals. Centre of mass. Moment of inertia. Theorems and examples on definite integrals. Eulerian integrals. Integration of ordinary differential equations of the first and higher orders and its applications. Linear differential equations. Simultaneous differential equations. Integration in series and by means of definite integrals. Calculus of variations.

Analytical Solid Geometry [*First year* Mathematics, Astronomy, Theoretical Physics, and Experimental Physics.]—— Straight line and plane. Surfaces of the second degree.

Elements of the general theory of surfaces.

Projective Geometry [*First year* Mathematics]——Projectivity of the primitive forms of the first dimension. Curves of the second degree and of the second class (synthetic and analytic methods).

Elementary Mathematics [*First year* Mathematics]——Elementary algebra. Domain of rationality and divisibility (of integers, of rational integral functions of one variable and of many variables). Algebraic solutions of the general equations of the first, the second, the third, and the fourth degree. Sturm's theorem, etc.

Astronomy and Method of Least Squares [*First year* Mathematics, Astronomy, Theoretical Physics, and Experimental Physics]——Spherical trigonometry. Fixed stars. Sun. Calendar. Planets. Moon. Eclipse. Rotation of planets. Satellites. Comets. Atmospheric refraction. Precession and nutation. Annual parallax. Aberration. Theory of probabilities. Law of errors. Method of least squares.

Elements of Calculus and Differential Equations [*First year* Chemistry]——Limits, continuity, differentiation. Infinite series and products. Taylor's theorem. Maxima and minima. Theory of plane curves. Definite integrals, indefinite integrals. Applications. Integration of differential equations.

General Theory of Functions and Theory of Elliptic Functions [*Second year* Mathematics and Theoretical Physics, and *third year* Astronomy]——Complex numbers. Functions of a complex variable. Conform representation. Complex integration. Theory of residuum. Evaluation of some definite integrals, and other applications of complex integration. Continuity and multiplicity of a function. Theory of power series. Taylor's and Laurent's Theorems.

Elementary functions. Singly periodic functions. Doubly periodic functions. Hermite's Theorem. The functions $\wp$, $\wp'$, ζ and σ. Addition-Theorem. Multiplication. Jacobi's notations (Fund. Nov.) and functions. Applications to the theory of numbers. Modular function as a special case of automorph-function. Inversion of doubly periodic functions. Applications.

Differential and Integral Calculus and Differential Equations [*Second year* Mathematics and Theoretical Physics]——Fourier's series and integrals. Legendre's and Bessel's functions, and hypergeometric series. Total and partial differential equations. Differential equations of mathematical physics. Infinitesimal transformation and its applications to differential equations.

Differential and Integral Calculus [*Second year* Astronomy]——The same as the above with infinitesimal transformation and its applications to differential equations cancelled.

Theory of Numbers and Algebra [*Second year* Mathematics]——Theory of numbers:——Binomial congruences. Quadratic residues. Binary quadratic forms. Theory of ideals of quadratic *Zahlkörper*. Determination of class number. Algebra:——Determinants. Bilinear and quadratic forms. Elementarteiler. etc.

Higher Geometry [*Second year* Mathematics]——Homogeneous co-ordinates. Collineation and correlation. General theory of algebraic curves. Cubics. Algebraic surfaces.

General Dynamics [*Second year* Mathematics, Theoretical Physics, and Experimental Physics]——Vectors. Motion of a particle and of a system of particles. General principles. Forces acting on a rigid body and their equilibrium. Uniplanar motion of rigid bodies. Motion of a rigid body about a fixed point. Generalized co-ordinates. Small oscillations. Cyclic system. Hydrostatics. Hydrodynamics.

Applied Differential Equations [*Second year* Astronomy,

Theoretical Physics, and Experimental Physics]——Bessel's functions and spherical harmonics. Integral equations. Application of elliptic functions. Differential equations of mathematical physics.

General Theory of Functions and Theory of Elliptic Functions (continued). [*Third year* Mathematics and Theoretical Physics, the latter being optional]——Riemann's surface and its connectivity. Analysis situs. Elliptic integrals of the first, the second, and the third kind. Binary forms ——theory of invariants and covariants. Application of the same to the reduction of elliptic integrals. Hermite's transformation. Jacobi's principle of transformation. Numerical evaluation of the elliptic integrals. Abel's theorem and its applications. Transformation of theta-functions.

Theory of Differential Equations [*Third year* Mathematics]——Fuchs' theory of linear differential equations. Gauss' differential equation. Integration of partial differential equations. Weierstrass' method of the calculus of variations. Introductions to integral equation.

Theory of Numbers and Algebra [*Third year* Mathematics]——Theory of groups of finite order. Galois' theory of equations. Select chapters from higher arithmetic and algebra.

Higher Geometry [*Third year* Mathematics]——One or two of the different kinds of geometries, such as differential geometry (curves in space, curvature of surface, deformation of surface, etc.), non-euclidean geometry, descriptive geometry, etc.

Mathematical Seminary [*Third year* Mathematics]——Themes accompanied by hints, suggestions, literature reference and so forth, are given to students, and they are required to work out in the spirit of independent research.

In most cases, instruction is given by means of lectures. Very often, particularly in the first two years, lectures are accompanied by exercises which are usually held in the afternoon. Students are required to solve problems within the range of the subject-matter of lectures under the supervision of the instructor who gives these lectures. Hints or suggestions are given whenever it is deemed necessary to do so, or they are asked for. The solutions obtained are examined and criticized by the instructor who has the charge of such exercises.

Instead of solving problems, students are sometimes required to instruct themselves by reading books recommended by their instructor, in order to supplement the lectures which he gives. In the second and the third year, exercises are sometimes replaced by a small group of students reading in turn some portions of a standard treatise or some original article of an eminent mathematician, wherein they are assisted by their instructor, and sometimes by each of them writing an essay giving a discussion of some special problem.

Again, in some special subjects, students are sometimes required, instead of being taught by lectures, to read some treatises or papers of some eminent authors, in order to accustom themselves to understanding matters written in foreign languages, at the same time that they learn the subject-matter. In geometry and allied subjects, models are used and the method of constructing them explained whenever it is possible to do so without going into too much detail.

The library of the Mathematical Institute has a fairly complete collection of mathematical books and journals, to which students have free access.

Such are the methods of instruction in broad outline at the Science Faculty of the Tokio University. The methods of teaching in other universities do not differ much from this.

The achievement in any subject of any individual student during any one academic year is represented by the so-called "year mark" whose maximum figure is 100 and which is determined by examinations or by any other means the instructor may think fit. The regulations for examinations [pp. 178-9 of the Calendar for 1911-1912 just published] which give somewhat minute directions for determining the year mark, have not been strictly adhered to in actual practice since several years. The average mark of a student is determined by dividing the sum of the year marks in all the subjects by the number of the subjects. The condition for *pass* is more than 50 in each subject coupled with an average over 60. If a student does not come up to this standard, he is not allowed to go on to the next *year*. If this occurs twice in succession, he shall be dismissed from the Faculty. There are a few unimportant exceptions to this rule, which it may not be necessary to mention.

The account of the University Hall which stands above the Faculties, lies beyond the scope of this report.

Faculty of Technology

The following ten Courses, each of which extends over three years, are to be found in this Faculty:——Civil Engineering, Mechanical Engineering, Naval Architecture, Technology of Arms, Electrical Engineering, Architecture, Technology of Explosives, Applied Chemistry, Mining, and Metallurgy. Mathematics is taught in all the Courses except the three last named.

The time given to the instruction in mathematics is three hours per week during the first and the second term of the first year. The subject-matter of instruction may be approximately inferred from the following books of reference at present in use :——

J. Perry——Calculus for engineers.

J. Lüroth——Vorlesungen über numerisches Rechnen.

A. G. Greenhill——Differential and Integral Calculus.

P. Appell——Éléments d'analyse mathématique.

So much of differential equations as is given in Appell's Analysis [2nd edition, 1905] pp. 548-674, is taught, if possible, before the end of the first term, and their applications together with other branches are given in the second term. As to the method of teaching, an instructor gives a course of lectures either of his own or taken from some text-books, and the students take notes of the essential points in these lectures. At the end of each term, is held an examination, at which from four to six questions are set and the students are required to hand in their answers within from three to four hours.

Some of the subjects closely allied to mathematics are taught in the several Courses; e.g. in the Course of Civil Engineering, to applied mechanics are given 6 hours in the first term, 2 hours in the second term, and 3 hours in the third term of the first year; and in most of the other Courses, in which mathematics is taught, to dynamics is given 1 hour throughout the first year, and to applied mechanics 3 hours in the first term, 2 hours in the second term and 1 hour in the third term of the first year.

Faculty of Agriculture

In this Faculty there are five Courses, each of which covers three years, namely the following: Agriculture, Agricultural Chemistry, Forestry, Veterinary Medicine and Fishery. Mathematics is taught only in the Course of Forestry, in which 4 hours in the first term and 2 hours in the second term of the first year are assigned to "Higher Mathematics."

To this Faculty are attached several so-called "subsidiary courses," each of which extends over three years. The condition for admittance being the completion of the middle school course or its equivalent, these courses are of the grade of a higher middle school or even lower. In the subsidiary course of forestry, trigonometry and elements of analytical geometry are taught in the first year and "forest mathematics" accompanied by exercises is given in the second year.

The Kioto Imperial University

The Imperial University of Kioto consists at present of a University Hall and of four Faculties, viz. of Law, Medicine, Literature, and Science-and-Engineering.* Mathematics is taught only in the last named Faculty. In this University, the academic year is divided into two terms; the spring term begins on the 21st of January and ends on the 10th of July, and the autumn term extends from the 11th of July to the 10th of January following.

The condition for admittance is less strict than in the case of the Tokio University, in as much as graduates of such schools (including Higher Normal Schools) as are recognized by the Minister of Education as having a course similar to the course of the higher middle school, may be admitted to the Faculties in case of vacancies.

Faculty of Science-and-Engineering

The Courses of instruction established in this Faculty are as follows:— Mathematics, Physics, Pure Chemistry, Chemical Technology, Civil Engineering, Mechanical Engineering, Electrical Engineering, and Mining and Metallurgy. Mathematics is chiefly taught in the first three Courses. Conditions for graduation are somewhat different in this Faculty from those in the corresponding Faculties of the Tokio University. A student has to study all the prescribed subjects of his Course in the interval of from three to six years.

*Here I thought, it would not be necessary to deviate from the official translation.

In the following, the mathematical subjects only in each Course and the number of hours per week assigned to each such subject, are given. Hereby, for the sake of convenience, we shall make use of the following abbreviations:—E stands for *exercise*, and M for *month*; a figure standing alone on the left the number of lectures and that on the right the number of years.

Course of Mathematics

	Hours per week	Duration
Analytical Plane Geometry [projective geometry, analytical and non-analytical].	2	1
Analytical Solid Geometry [co-ordinates, plane, straight line, conicoid, applications of differential calculus to solid geometry].	2	1
Higher Geometry Part I [differential geometry].	3	1
Higher Geometry Part II [algebraic curves, cubic curves, quartic curves].	3	1
Differential and Integral Calculus, and Theory of Differential Equations [functions of real variables, differential co-efficients, applications, indefinite integrals, definite integrals, applications, ordinary and partial differential equations].	5 6 E	1
Theory of Functions Part I [theory of functions in general, theory of elliptic functions].	3	1
Theory of Functions Part II [theory of special functions, algebraic, automorphic, hypergeometric and allied functions]	3	1
Higher Algebra [theory of groups, Galois' theory of equations].	3	1

	Hours per week	Duration
Theory of Numbers [congruence, quadratic rest, binary quadratic forms, distribution of primes, cyclotomy].	3	1
General Dynamics Part I	4	1
General Dynamics Part II	3 E.	6 M.
General Physics Part I General Physics Part II	4	1
Elements of Astronomy	2	1
Physical Experiments	6	1
Special Lectures	3	1

Course of Physics

Analytical Solid Geometry	2	1
Differential and Integral Calculus and Differential Equations	5 6 E	1
Theory of Functions Part I	3	1
Theory of Errors	2	4 M.
Theory of Measurements	2	6 M.
General Dynamics Part I	4	1
General Dynamics Part II	3 E.	6 M.
Special Dynamics Part I [principles of dynamics]	4	4 M.

Course of Pure Chemistry

Either the one or the other of the following two, to be taken at the option of a student.

Differential and Integral Calculus, and Differential Equations	5 6 E	1
Integral Calculus and Differential Equations	3	4 M.
	2 E	6 M.

Besides the Courses which have been already noticed, mathematics is taught in the Courses of the Civil, the Mechanical, and the Electrical Engineering. The subjects are Integral Calculus, Differential Equations, and Dynamics. The time allotted to them is insignificantly small as in the case of the corresponding Courses at the Tokio University.

The conditions for graduation are very similar to those at a German university, the graduation examination here corresponding to the *Doctor Examen* there. A student *must* stay in a Course at least three years during which he *can* finish it under normal circumstances. He is allowed to stay three years more but not longer. The completion of an individual subject often tested by an examination is certified by the instructor in the subject. When he has completed all the prescribed subjects in a Course, whereby one or two special subjects may be excepted, in time not less than three years, he reports himself for the graduation examination. For full particulars, reference is to be made to the regulations to be found on pp. 94-5 of the latest [1911-1912] Calendar.

As the very name Science-and-Engineering or Science-and-Technology would suggest, in establishing this faculty, an experiment was made to bridge over the gulf usually found between science and technology by their co-habitation in one and the same faculty. That the actual result has not come up to the exalted expectation of those who had hit upon the idea, is, I believe, tolerably well acknowledged. I should have liked so much to say something on this topic. But time is pressing and I have to hurry on to saying a few words about the conditions of graduating a Course in this University. They are the imitation of those in vogue in many of the German universities, modified so as to suit, so to say, *local* conditions. So far as concerns Courses in pure

sciences, absolutely speaking, I, for one, do believe in the, by far conspicuous, superiority of the usage similar to that in a German university. But whether this is compatible with the deep-rooted *local* conditions, and, if incompatible, whether we can change or modify these irrational *local* conditions, are difficult problems to solve. Here again I wish to be allowed to add some words on this question, upon which I have concentrated the thoughts of my leisure hours for many years past. But time is ever more pressing, and I have to leave, though ever so reluctantly, all these topics to some future opportunity which so rarely presents itself.

The Tōhoku and the Kiushiu Imperial Universities

There still remains the giving an account of two more universities, namely, the Tōhoku and the Kiushiu Universities. But they have been only recently founded and even the Faculties already in existence are not in full working conditions; so we shall confine ourselves to giving but a meagre delineation of a few salient features of these institutions.

The Faculty of Science of the Tōhoku Imperial University was opened in the beginning of 1911 and the actual work of instruction begun in September of the same year. It contains three Courses of Mathematics, Physics, and Chemistry, to which the Course of Geology will be added in the near future. As regards the division of the academic year into three terms, the duration of a Course, and the subject-matter of instruction in different Courses, as well as in very many other respects, it is not much different from the corresponding Faculty in the University of Tokio. In one important respect, however, it does differ, owing perchance to unavoidable circumstances, not only from its sister Faculty but from most of the Faculties of other Universities. Namely, the conditions for admittance have been made less strict to such a degree that it is causing an anxiety to all those who have the sound growth and the future prosperity of the budding institution sincerely at heart.

Some of the *names* of the subjects in the curriculum of the Course of Mathematics in this Faculty, which might catch the eyes of a casual observer accustomed to seeing the curriculum of the same Course in the older universities, are: in the first year, Algebraic Analysis (determinants, continued fractions, infinite series and products, etc.) and, as an

optional subject, Descriptive Geometry; in the second year, Synthetic Geometry, and as an optional subject, Practical Mathematics; and in the third year, Differential Geometry, and as optional subjects, History of Mathematics and School Mathematics.

The Faculty of Agriculture of the Tōhoku Imperial University as the continuation of the Sapporo Agricultural College has an illustrious past, intimately connected with the development of natural resources of Hokkai-Dō, of some four decades, its seed having been sown in the very year when the famous First Educational Code was promulgated. At present, it contains four Courses of Agriculture, Agricultural Chemistry, Forestry, and Veterinary Medicine. Forest mathematics and method of least squares are taught in the Course of Forestry. A unique feature of this Faculty is that it has a preparatory course of its own, corresponding to the higher middle school course and not much different from it at least in form. Besides, it has subsidiary courses of agriculture, forestry, civil engineering, and fishery.

The Kiushiu Imperial University consists at present of the two Faculties of Medicine and Technology. The Faculty of Technology contains six Courses of Civil Engineering, Mechanical Engineering, Electrical Engineering, Applied Chemistry, Mining, and Metallurgy. Mathematics coupled with Dynamics is taught throughout the first year in the first three Courses 3 hours per week and in the last three Courses 2 hours a week. A fact worthy of notice is that this Faculty of Technology, unlike its sister faculties, has an independent Chair of Mathematics and Dynamics.

From the beginning, I had not the slightest intention of giving even a rough idea of the University Halls. So I

have now done, though very unsatisfactorily, with the Universities.

It would be a matter of great regret not only to myself but to many of my friends and colleagues, were I to let this opportunity pass without acknowledging the indebtedness which the study of higher mathematics in our country, which does not date much further back than two decades, owes, though indirectly, to some of the mathematicians of world-wide fame, among whom should be particularly mentioned L. Kronecker, E.B. Christoffel, Th. Reye, K. Weierstrass, I. L. Fuchs, H. A. Schwarz, D. Hilbert, H. Weber, F. Klein, F. G. Frobenius, and O. Hölder.

What is given in the following, was originally intended for insertion in the chapter which is to treat of the teaching of mathematics in technical colleges and schools, but is, for reasons to be hereafter explained, appended to this chapter.

Concerning the teaching of mathematics in technical education, there seem to be two opposite points of view. I will not say *diametrically opposite*, though they may appear so at first sight, as I wish to leave open the chance that when everything has been cleared up, they may coincide after all in spite of their incipient appearance.

One point of view is that which may perhaps be characterized by such a phrase as "practical in the extreme" or, in somewhat elaborate language, "ultilitarianism *extempore*." This point of view has been so often and so dexterously unfolded that it would hardly require any further elucidation. It was even so humorously illustrated that boys of mathematical inclination were compared to ducklings skilled in swimming.

The other point of view has the disadvantage of being so difficult to be explained in such a humorous way

as in the tale of ducklings, which appeals so strongly to the concurrence of the larger public. For this point of view, it was a misfortune that it has not received the support of some one who could invent such an interesting story as that of the swimming ducklings. I cannot do better than quote what has been often quoted here in Japan since the day when Professor Ostwald, one of the greatest thinkers of modern times, uttered the following words in opening his address at the second anniversary meeting of the German electricians on June 7, 1894:....."The would-be practical Englishman with the intention of entering a dye-factory, studies, not general chemistry, but the chemistry of dye-stuffs. The German studies chemistry, lock, stock and barrel, never wrecking what his calling is to be. Only when he has a really scientific foundation, will he begin to build up his special knowledge. By and by there comes a change over the face of the industry in which these competing chemists are employed. The German——he is ready; without difficulty he adapts himself, and follows up the novel course. But the Englishman——he can not imagine at what position he has arrived; he must begin, so to say, over again."

The actual conditions in the two countries might have changed since then; but the truth embodied in the above words is as fresh to-day as it was then and applies to the teaching of mathematics in technical education with even greater emphasis than to the teaching of chemistry. The world we live in moves incessantly while we are having such difficulties in following things moving and are prone to stop where there is no halting place. In every case, the teaching of mathematics in technical education must be practical. Only there is this difference: ultilitarianism in the sense of being useful instantaneously but once for all, and utilitarianism having the potentiality of manifesting its usefulness whenever there is a necessity.

If the instantaneous utility is the desideratum, there is no use of teaching mathematics beyond showing how to calculate numerical values of ready-made formulæ so conveniently packed in the "Practical Engineer."

The public which has no distinct conception of its own school days, is cruel in demanding that those fresh from some technical schools should prove to be useful all at once in a factory. Food must be cooked before it can be eaten. Ready-cooked food will soon putrefy.

But then, are the two points of view mentioned above irreconcilable? Now there are many grades of technical schools; at the top the faculties of the Imperial universities, then technical colleges, then technical schools of the middle school grade, and then technical schools of lower grades. If one can get a glimpse of the truth underlying this grading, then and there, the two points of view are already reconciled to his utmost satisfaction. Again in a country like ours, where excessive centralization takes place in everything pertaining to education, there are chances of the concentration of the best youths. They can be taught without being taught, and the experience thus gained is not the safe guide in leading us through the complexities of technical education.

CHAPTER IX

(The last chapter)

FRAGMENTS

I have now done with the teaching of mathematics in the main trunk of our *educational tree*. There still remains for me to deal with some of its more important branches. As originally planned, there were to be five or six more chapters, namely, one chapter each dealing with female education, normal school education, industrial (commercial, technical and agricultural) education, recruitment of middle school teachers, and miscellaneous schools respectively, and perhaps one more chapter on the teaching of mathematics in Taiwan (Formosa) and Chōsen (Korea).

As I am writing these very lines, the time of my departure for England to join the Cambridge Congress, which has been fixed for the middle of June is drawing near——almost in sight. Moreover I have already accepted the flattering offer to represent the Imperial University of Tokio at the coming celebration of the two hundred and fiftieth Anniversary of the Royal Society of London, which takes place on the 15th of July. Thus I cannot even postpone my departure. Anyhow I have to stop writing so that there may still remain time for finishing the printing.

There are some, so to speak, crude materials which I have been collecting ever since last autumn and which I intended for insertion in the chapters which were to follow. A few of them are given here as fragments. Among these fragments there are some which have nothing to do with the teaching of mathematics. They were in the first instance intended for preludes to something mathematical. But now having been deprived of their sequel and standing isolated, they have lost their *raison d'être*. So, I might just as well

have omitted them altogether, if it were not for that they have, so far as I am aware of, never appeared in print. For this as well as for many other incongruities, I crave for indulgence of the reader.

For preparing teachers for elementary schools, there are normal schools [for men] and normal schools for women. Sometimes the one for women is attached to the one for men. Each prefecture must have at least one normal school, provided with an elementary school for the training of pupils in the methods of instruction. To normal schools for women should be attached kindergartens whenever possible. The condition of admittance is the completion of a two years' course of higher elementary course or its equivalent, and the regular or main course of study extends over four years. Thus normal schools are a branch of our *educational tree* which are connected by the intervention of higher elementary schools to the main trunk at the point of division between the elementary school and the middle school.

According to the statistical return for the year 1908-9, the total number of normal schools was 75, of which 32 were for male pupils and 23 for females, while there were 20 for both male and female. Here in our country, as in many other countries of the world too, I believe, we are experiencing the scarcity of well qualified elementary school teachers, notwithstanding that the supply of such teachers are not necessarily confined to the product of normal schools.

In connection with the teaching of mathematics in normal schools for women, we meet with a problem of "just a little bit" of algebra or geometry. The same problem presents itself very often and in very many places. A little bit of geometry is as difficult as, perhaps even more, than a little bit of calculus. On this little bit, more has been written than on any other bit of mathematics.

If it is merely for the sake of the name or show that something called geometry has been taught or learned, there remains nothing more to be said. On the part of those who are going to revise the curriculum of one school or another, they demand the insertion of a little bit of geometry without thinking of whether such a demand is admissible or not, not to say of its great difficulty, and think that great progress has been achieved by putting it in a place where there has been hitherto no such little bit.

The problem of "a bit of geometry," I believe, is not an old one, and really dates from the days of Paul Bert. The possibility of such a little book is so ably treated in the author's preface of "Elementary Geometry" by Olaus Henrici, late Professor of pure mathematics in University College, London, who was an old teacher of mine for but a short time. This book was translated into Japanese and published at a date not far from when the first edition of the original was published in 1878. Ever since, many attempts were made by several of our teachers of mathematics to produce a little bit geometry which should adapt itself to our school conditions. Many of the artifices such as paper folding, to be usually seen in kindergartens, have been tried. Much will depend upon whether a little bit of geometry is to be followed by geometry proper or not. So, uniform treatment of this little bit is out of the question. However interesting and even fascinating some of these treatments are, I cannot go into detail now. I may only add that the "First Book of Geometry" by G. C. Young and W. H. Young, of Dent's Series of mathematical and scientific text-books for schools, and its German translation "Der Kleine Geometer," is much appreciated here.

Having just spoken of normal schools for preparing elementary school teachers, by way of continuation, we will

now speak of Higher Normal Schools, and, in the first instance, of those for men. There are two such schools, one in Tokio and the other in Hiroshima. The Tokio Higher Normal School is an old institution and has passed through various stages of metamorphosis and development. Towards the close of last year, as was said before, it commemorated its forty years' existence. To begin with, it is a school for preparing normal school teachers. Since many years there have been inward tendencies to extend the function of the school to preparing middle school teachers. Indeed this tendency is now being persistently and vigorously followed.

The Tokio Normal School which was the name given to the germ of the present institution bearing the same name with "Higher" inserted into its middle, was founded in 1872—3. To the development at the initial stage of this school, few seem to have contributed more than two Americans, M. M. Scott and Dr. David Murray, the former directly by actually teaching at the school and giving useful advice on the spot, and the latter indirectly, his position at that time being the Adviser at the Department of Education. Their connections with the teaching of mathematics has been already mentioned elsewhere in this report. An account of their work while in our country will be given in the appendix to this Report.

There are two Higher Normal Schools for Women, namely, one in Tokio and the other in Nara. To the former which is the much older institution of the two, we shall refer later in speaking of female education.

We will now speak a little about the recruitment of middle school teachers. I do believe that how to recruit candidates for middle school teachers from among our most promising young men is a question far more important and difficult than the preparation of such teachers.

Generally speaking, the profession of a teacher is a laborious one and by no means lucrative. He has to wait long before his labour bears fruit. For him it is almost forbidden to make a name for himself or to shine in the eyes of the world. Teachers are,——if not, ought to be—— deeply absorbed in their daily work so that they have no time or inclination to step outside their profession and endeavour to acquire a position in the general estimation. Indeed it is questionable whether they would ever receive the consideration due to their noble work in the circles where power and influence reside, except perchance through the channel of some influential men having their own boys at middle schools. Men who are willing to sacrifice themselves for the sake of the younger generation or find their own pleasure in such work, without heeding their former class-mates who, having chosen different careers, might perchance have attained to prominence in politics, in the business world, in the medical or engineering profession, and the like, are very difficult to find in these days of instantaneous utilitarianism, when any benefit however significant but indirect is required to bow before any other benefit however trivial but direct. Fortunately, however, men are numerous and their inclinations as varied as they are numerous. The best thing we can do under the above inevitable conditions, is to make and keep the entrance to the teaching profession as wide as possible. Let as many teachers as possible be recruited from the largest public, so to say, if such an expression is allowable, and let the principle of the survival of the fittest find its application here. That is the best means of promoting our middle school education.

Higher Normal Schools should serve for one of many, but not the only, ways of supplying middle school teachers, side by side with their specific function of supplying teachers for normal schools. Our normal school education was modelled on the American system and has not changed its

character ever since. I am told that, even to this day, the business opportunities in America are so great that it is hard to see how as many men of high intellectual attainments should be coming into the teaching profession as are to be found in the more densely populated countries. I believe, the American system of normal school education was originated, and has since developed, at the time when the scarcity of teachers' candidates arising from the above mentioned conditions was far more keenly felt than to-day. That we have selected America from among so many different foreign countries to serve for our model in framing our normal school education, was a wise policy on the part of the pioneer workers in the field. The conditions then prevalent here were very much like those which prevailed at that time and, as we are told, prevail to a certain extent to this day in America, although they were, it needs hardly mentioning, due to entirely different causes——during the life-time of the oldest of us we never had occasion to complain of the scarcity of our population. The same conditions hold to-day even more emphatically as regards the elementary school teachers. The increase of the number of elementary schools has far outstripped the growth in the supply of teachers for such schools, and as ever we are complaining of the lack of well-qualified elementary school teachers. But it is otherwise with the middle school teachers. There are nowadays so many quarters, toward which we may look forward for the recruitment of efficient middle school teachers.

The imperialism and consequent expansion of normal school education in the direction of monopolizing the supply of every kind of teachers, have gone to such an extremity that arguments as shallow as "because one is to become a teacher, he *must* receive normal school education" are offered in broad day-light to the amateur public that neither knows nor stops to think. On the other hand, there is a human weakness which manifests itself in seeking uniformity in

anything for no other purpose than for the sake of convenience in handling or registering. Indeed it would be exceedingly convenient in this sense, if all the middle school teachers were supplied by schools of one kind. Again, whenever there is anything to be discussed concerning middle school education, advice is usually sought of the middle schools attached to the Higher Normal Schools, in absence of any organization which represents a large number, if not all, of the middle schools. All these various causes are constantly at work in contributing toward tightening the grip more and more, with which our middle school education is held by our normal school education.

In order to avoid misunderstanding, I should like to use, just for the present, the phrase "training in the art of teaching" in a sense quite distinct from normal school education. As was said before, let the entrance to the teaching profession in middle schools be made and kept as wide as possible. Let us wait as long as possible before our young men make up their mind to go into the teaching profession of their own free will after having become conscious of their inclinations in this direction. Let such young men come forth from all available sources. To them who are already equipped with necessary knowledge and for whom there is not much fear that their inclination will change in the near future, let us give *training in the art of teaching* for a short period of, say, one year or even only six months.

In the case of normal school instruction, our boys fresh from middle schools and with their inclinations oscillating to and fro, are asked whether they would go into the teaching profession in middle schools, at the time when they might be found repenting their past deeds of having outwitted their former teachers. There are some noble exceptions. But, generally speaking, it is due either to some sheer necessity or to some irony of chance, that boys make up

their mind to go into the teaching profession. It is true that, among their comrades, there are graduates of local normal schools. But then, they too had to pass the same crisis and, in fact, very much earlier. To the aggregate of such boys is given *normal school education* which extends over many years.

I am afraid I have not as yet made quite clear what I wished to explain. But I must hurry on. So, without any more preliminaries, I hasten to explaining how our middle school teachers, including, of course, those of mathematics, are recruited from various sources.

One who wishes to become a regular teacher in a middle school must have a teacher's license given by the Department of Education, in view of his qualifications. In passing we may observe that there is no such restriction in the case of higher middle schools.

Teacher's licenses are given without examination to the graduates of Higher Normal Schools and the so-called Temporary Institutes for the training of teachers, of which there are several, all attached to some permanent schools. Teacher's licenses in one or two special subjects are also given to the graduates of various Courses in the Imperial universities. There are some other qualifications which entitle the qualified to receiving teacher's licenses without examinations, but they are of minor importance as their number is comparatively very small. The Temporary Institutes are, as the name indicates, of transitory nature; so they may also be left out of consideration. Now, it is often said that a higher normal school is an institution established by the Government for the express purpose of preparing teachers and therefore its graduates *must* receive teacher's licenses. On the other hand, notwithstanding that a large number of the graduates of the faculties of literature and of science actually goes into teaching profession, it is only allowed to say that they *may* get teacher's licences.

It is this difference in tone which is worthy of profound meditation. Both the graduates of a higher normal school and of a university can get teacher's licenses without examinations. But, so to speak, the former get them as their right and the latter as a favour.

There is the so-called "Department of Education's Examination for Licenses to Teach in Intermediate Schools," which takes place once every year. I should have liked to say something about this examination, as I have first-hand knowledge of it, having served on its Committee for so many years. But now I must confine myself to bare essentials. The preliminary qualification is the completion of the middle school course or its equivalent or something very near to it. Applicants are examined by an examination committee specially appointed for the purpose. Those who have successfully passed the examinations are entitled to teacher's licences. This Examination which has a history that goes back to some twenty-five years ago, has proved itself to be exceedingly useful in very many respects. For a long time to come, it will exist, and probably it will never be abolished. But, in theory, it is of a temporary nature under the dogmatic principle that the only proper channel leading to the teaching profession should be through the normal school education. Now I have been thus far trying to explain the tone and not the substance. I thought it necessary to do so, as it is often the tone which appeals to the larger public and sometimes decides the destiny of the substance.

We will now give just a meagre account of how examination in mathematics for teacher's licences is carried out. A special examination committee is annually appointed from among the professors of mathematics at the Imperial universities, higher normal schools, higher middle schools, etc. There are four grades of examination, namely,

(*1*) Arithmetic, algebra, and geometry;

(2) Trigonometry;
(3) Analytical geometry;
(4) Differential and integral calculus.

Candidates are examined in different grades in the above order successively; but they are not required to pass through all the grades in any one year.

The examination in the first grade consists of a preliminary examination and a main examination. The preliminary examination is conducted at the local prefectural offices, the printed copies of the examination questions being sent to these places. Only those who passed this examination are allowed to go for the main examination which takes place at the center i.e. in Tokio. The main examinination consists of two kinds, namely written and oral. The written examination may, in a certain sense, be looked upon as a continuation, or, perhaps it would be more appropriate to say, a verification, of the preliminary examination. As regards the oral examination, in algebra and geometry, candidates are examined socratically in the subject-matter, while, in arithmetic, they are required to show how they can teach. We may also add that they have to go through a written examination in the rudiments of pedagogics.

The examinations in the second and the higher grades take place in Tokio; and they are all written examinations. The standard of attainments tested is nominally, in conformity with the *tone* explained above, that corresponding to the higher normal school; but practically it has been fixed by the long continued experience of the Examination itself. Those who have passed through the first grade are already entitled to the license and can become regular middle school teachers. And their further successes in the examinations of higher grades do not add to the official qualification as teacher, but only increase the estimation in which they are held by the school authorities. I have, however, been told

that, in schools under the Navy Department, the passing of all the grades has been made the qualification for becoming regular instructors in such schools. The Examination has rendered an useful service to the State. It is a well-known fact that a large number of our best middle school teachers, particularly those in mathematics, has been derived from this source. Besides, it has exercised and is still exercising a powerful influence in stimulating the study of mathematics outside the circle of school teaching. I see no reason why this Examination should not celebrate its twenty-five years' existence, in view of the large number of such celebrations which are too common nowadays.

Many years ago, it was argued: Arithmetic, algebra, geometry, and trigonometry are the only subjects to be found in the middle school curriculum; why should the candidates for teachers in such schools be examined in subjects other than these? The wonderful feature of the argument found its climax, when it was asserted that in *principle* (?) at least the examinations in subjects beyond those actually taught in middle schools should be abolished. The power which can grip the throat often yields to the *tone*, of which I have already spoken. And analytical geometry and differential calculus were actually struck out from the list of examination subjects. But, as a matter of course, they were soon reinstalled in their original position. I thought it necessary to say this, even while I am in such a hurry, so that the same experiment may not be tried again.

Under the Department of Agriculture and Commerce, there is an institution called the Place for Marine Products. There is an interesting story about the name of this———I do not know what to call it. At one time, when the tendency of centralization in everything pertaining to education was

at its zenith, anything called *school* had to fear its being absorbed by the Department of Education. On this account the said institution, although it has really the nature of a school, was purposely called *place* and not *school* in defiance of many inconveniences arising therefrom. I cannot vouchsafe for the accuracy of the anecdote. Were it an invention, so much more it would illustrate to what extremity this spirit of centralization has gone in past days. It reigns still dominant, only perhaps not to such a degree as in former days, there being less necessity after having already effected its purpose in centralizing everything which can be centralized. To understand clearly the educational matters in our country, it is of utmost importance to know of this spirit of centralization not only for foreigners but also for our own countrymen who are too young for having been eye-witnesses of the very many changes and reforms and whose intelligence is handicapped by contact with the happenings of the hour.

Among seemingly harmless evils, which are too common nowadays, I should like to notice one more out of so many of them. I mean the evil of the so-called "teaching by correspondence," which has of late come in vogue. It is even hailed as a great invention of modern times! However innocent, even useful, it may appear on its face, teaching by correspondence, as is usually done here, is pernicious in as much as it dissipates the earnestness and annihilates for ever that spirit which is absolutely necessary in *learning* anything in the true sense of the word. How far the evil has gone as regards the teaching of mathematics in particular, may be inferred from the advertisements, to be often seen in daily papers, alluring our young folk to "mathematics taught with the *greatest rapidity*." In educational matters we feel more and more the need

of progressive conservatism which takes intelligent notice of everything going on around and even forestalls the coming evils of clap-trap innovations or reforms.

I now wish to say a few words concerning our female education. An event which marks an epoch in the initial stage of the occidentalization of our female education was the sending in 1871 of five young Japanese women of rank to America in care of Mrs. De Long. The girls referred to were Misses Y[oshi] Yoshimatzu, age fifteen, T[ei] Uyeda, fifteen, S[utematsu] Yamagawa (now Princess Ōyama), twelve, S[hige] Nagai (now Baroness Uriu), ten, and U[me] Tsuda, eight.

The first girls' school properly to be so called, named the Tokio Girls' School, was definitively established by the Department of Education in 1872 in the site where the Department itself is now domiciled. The instruction given seems in substance to have been something like what is now taught in elementary schools with predominance of the English language. Among the teachers were Mrs. Veeder, Mrs. Rice [probably wife of W. S. Rice, then Interpreter in the United States Legation in Tokio, who accompanied the memorable Iwakura Embassy of 1872], Miss Griffis, and Miss Washington [probably the one who later became Mrs. Ewing]. The school was discontinued in 1877, but it has left a lasting influence upon the development of female education in our country. Indeed this school may, in a certain aspect, be looked upon as the germ of the present Tokio Higher Normal School for Women. About this time there were also two more schools of a similar kind known by the name of Kaitakushi Girls' School and Kōbushō Girls' School; but they have left but little trace beyond their names.

Quite recently, a new course of dealing with female education was inaugurated by establishing a new kind of girls' high schools of slightly lower grade than the ordinary girls' high school, in which the consideration that the majority of pupils will some day become house-wives of the social middle class, is persistently kept in view. It was a difficult virgin problem for me to find a suitable English name for this kind of girls' schools. After much hesitation, I came to suggest the name of "girls' high real-school," which was adopted in the Divisional Report dealing with such schools. According to the official return made in April 1912, there were 204 girls' high schools, of which 2 were government institutions, 151 public, and 51 private, and 55 girls' high real-schools including 46 public and 9 private institutions.

As was said before, to normal schools for women are usually attached kindergartens. Some years ago, I took a fancy of observing how small children are taught in primitive counting. And I should have liked to say something concerning the *teaching of mathematics* in kindergartens, if not for lack of time. But I can now add only the following unfinished translation of a few action-songs used in our kindergartens.

Unfold your swelling buds round and full;
One, two, three, four, I will take you all;
I'll take you home in one big, pretty bunch.

Here is another.

Out in the sunshine the flowers are come;
Camelia, Camelia, dressed in red and white;
Dressed in white and red, each one and all.

Besides the various kinds of schools mentioned either in this report or in the Divisional Reports, there is a large

number of miscellaneous schools. Of these, I can now give the names only of just those which happen to cross my mind at this moment.

The Nobles' School.

The Peeresses' School [now incorporated in the school just mentioned].

A large number of Mission Schools both for men and for women.

Quite a number of the so-called Private Universities, one of which contains a section of science-and-technology.

Various kinds of medical schools.

Agricultural schools of the middle and lower grades.

etc. etc. etc.

Not only in this chapter but throughout the whole of this report, there have been many topics which I could but lightly touch, many others which my poor command of the language has forbidden me to explain clearly, and there remain so many things yet to be noticed. But I *must* stop now. Farewell! Only I cannot help looking back and say once more: *Adieu! Au revoir!*

APPENDIX

SKETCH

OF

DAVID MURRAY, PH. D., LL. D,

AND

A LETTER

FROM M. M. SCOTT TO R. FUJISAWA

Sketch of David Murray, PH. D., LL. D.

David Murray was born October 15, 1830. His parents were Scotch, of the old Murray clan which occupied Perthshire. They came to America in 1818, and joined the Scotch colony which was already numerous and thriving on the upper tributaries of the Delaware river.

He together with his elder brother, the late Judge William Murray of the New York Supreme Court, was educated at the Delaware Academy at Delhi. David early exhibited a great desire for knowledge. He learned to read while lying in his little crib when five or six years old, suffering for more than a year from a serious accident. He prepared for college at the Fergusonville Academy, and entered the sophomore class of Union College. He graduated in 1852 with a high reputation for scholarship. Dr. George H. Cook, who at that time was principal of the Albany Academy, went to Union College for an assistant. David Murray was recommended, and thus his lifework commenced. He served first as an assistant, then as professor of mathematics, and in 1857 was appointed principal of the institution, which under his charge attained a high state of prosperity and efficiency. He received the most flattering testimonials from the trustees on leaving his position in 1863 to become Professor of Mathematics and Astronomy at Rutgers College.

In 1872 an important embassy was sent from Japan to establish relations with foreign countries and to study the institutions of western civilization. It was especially charged with the duty of investigating foreign education and of devising plans for Japan. Many Japanese young men had been sent to Rutgers College as students. Professor Murray took a deep interest in them and they were all constant and

ever-welcome visitors at his house. The members of the embassy in the task of studying and collecting educational information called Professor Murray to their aid, and so satisfactory and efficient were his ideas of their needs that he was invited by them to go to Japan to assist the government in introducing a national system of education. He went in the spring of 1873 and remained in the service of the government until the winter of 1879. His official position there was Adviser in Educational Affairs. His work consisted in advising and aiding the Department in establishing a system of education. He was able to see before his departure this great work in successful course of fulfilment. In connection with the Centennial Exhibition at Philadelphia, Professor Murray returned to the United States as an accredited representative of the Japanese Department of Education, and spent a year in collecting information and material for the museums of the Department. He spent a great part of the winter of 1875–76 in Washington, at which time he prepared and published an open letter to Congress and a pamphlet on the subject of the restoration of the Japanese Indemnity Fund, $ 750,000. Later this indemnity was returned to Japan. He left the service of the Japanese Government in the winter of 1879, receiving every mark of respect which the government could bestow, and was decorated before leaving with the Third Class Order of the Rising Sun.

Returning through China, Egypt and Europe, he arrived home in the autumn of 1879. In December of the same year he was appointed Secretary of the Board of Regents of the University of the State of New York. This position he held until May, 1889, when after a very severe illness caused by the impure ventilation of his office in the Capitol at Albany, he resigned and returned to New Brunswick, New Jersey, where he had always maintained his house.

It was impossible for a man who delighted in work to

retire from activity and happily he was soon engaged by the Messrs. Putnam's Sons to write for their "Series of the Story of the Nations" their volume on Japan. He had brought a large amount of material on various subjects from Japan, and he entered into the writing of this book with the greatest zest. Visiting various libraries, particularly the Congressional Library at Washington, studying there the diplomatic correspondence, and receiving new material from Japan, he devoted his energies in making this book as full and trustworthy as it was possible to do.

In 1873 he published in Scribner's Magazine a popular exposition of the Transit of Venus, and in 1874 at the Transit of Venus he was with Professor Davidson and party at Nagasaki, Japan, when making their observations. His Manual of Land Surveying was published while he was Professor of Mathematics in Rutgers College. He prepared for the Philadelphia Centennial Exhibition the volume on Japanese Education. He gave lectures on Japan at Union College in 1881, and at Johns Hopkins University in 1897 and 1899. He wrote for the extensive book on the Public Service of the State of New York that portion referring to the organization and work of the Regents, and for the Bureau of Education at Washington the "History of Education in New Jersey." He contributed to and edited the "History of Delaware County in New York." He also wrote for the Historical Association an article called the "Anti-Rent Episode" in New York, and various other monographs and papers.

At the request of President Raymond of Union College he gave a very short account of the Development of Modern Education in Japan. It has been his intention to write a book on this subject, giving a full account of the work accomplished during his residence in Japan down to the present time; but on his return to New Brunswick he was called upon to take part in the public and charitable affairs

of the town, and therefore this work, which would have been both useful and interesting, was not accomplished.

Dr. Murray was for some time Trustee of Union College, and the Albany Academy, and Trustee of Rutgers College at the time of his death. He received the degree of Ph. D. from the University of the State of New York in 1863, that of LL. D. from Rutgers College in 1873, and LL. D. from Union College in 1874. He married, in 1867, Martha A. Neilson, of New York City.

It was a disappointment to him that he was not spared to finish for the new edition of the "Story of Japan" the history of the Russo-Japanese war. He had been occupied during the last year in preparing himself for this work. Baron Kentaro Kaneko, LL. D., has promised, however, in memory of the work Dr. Murray had done for his country, to contribute an addition to the new issue of the book, and this with the thoughtfulness of the Japanese Minister at Washington, who sent a representative of himself and the Japanese Government to the funeral services of their late educational adviser, are very appreciative tributes to his memory.

He died on March 6, 1905. His death came as a shock to his friends, and was felt throughout the town where he had spent the last fifteen years of his life and ten years at an earlier period.

He was a man who, wherever his residence might be, made himself felt in the community as an influence for good. He was not a great talker, but the word of encouragement fitly spoken where it was needed, of appreciation of work well done, of counsel to the student, was never wanting, as the numerous testimonies since his death give evidence with a most pathetic tenderness. He was a wise and calm and self-reliant man, eminently modest, not elated by success or disturbed by failure.

How much his services while in Japan was appreciated and admired, may be seen from the following extract from

the speech delivered by Fujimaro Tanaka, afterwards Vicomte Tanaka, at that time the acting Minister of Education, at the fairwell banquet given in honour of Dr. and Mrs. Murray, December, 1878:

"Five and a half years ago Dr. Murray entered on his duties in the Department of Education. During the entire term of his engagement without the least interruption he has filled that office in the most faithful and efficient manner. It will be interesting if I enumerate some of the most important and conspicuous results of our educational progress accomplished during his term of service. First, I will mention the development of the Tokio University, which from a small beginning in 1873 has been brought to its present efficient condition and thus the foundation laid for higher education in our country. Next, the establishment of the Female Normal School and the Kindergarten has been accomplished, and thus female education has received a great impulse and the training of small children in schools introduced. An educational museum has been established and a great variety of valuable and instructive material collected, and the means thus provided for showing our people the educational methods of other countries. All these great works have been accomplished by his judicious advice and his valuable coöperation. Besides all this I must not omit to mention the fact that the establishment and improvement of the regulations in regard to courses of study in the schools and colleges under the control of the Department have been mostly effected through his counsel."

"In regard to the intercourse so long maintained between Dr. Murray and myself, I can only say that both in official and private relations, it has been most satisfactory. His mild and graceful speech have left in my heart most delightful impressions. The brilliant results he has achieved in his service here will continue to be felt in their influence on every literary and scholastic interest. They will shine on

our Oriental Empire with the same brilliancy as the splendid decoration which His Majesty the Emperor has bestowed on him for his special services."

Soon after her husband's death, Mrs. Murray made a gift of one thousand American dollars to the Imperial University of Tokio, for the purpose of founding a prize for mathematics in memory of her husband. The gift was accepted by the University, and the prize, to be called "The David Murray Memorial mathematical Prize," was instituted, subject to the following regulations:—

1. The prize, provided out of the income of the above fund, and accompanied by a certificate, shall be awarded annually by the President of the University to the student in mathematics of not less than two years' standing who shall be judged most worthy of receiving the same by the Professors of Mathematics.

2. The prize may be given in money, or in any other form the Professors of Mathematics may recommend.

3. No student shall be awarded the prize more than once.

4. If in any year no student be deemed worthy of the prize it shall be withheld.

5. If there be a reserve surplus of the income of the fund, arising from non-award of the prize, and if more than one student be found worthy of the prize, then, two prizes may be awarded in any one year, but not more.

6. The prize may also be divided between two students, if both be judged equally worthy.

On April 20, 1908, Dr. David Murray Memorial Dinner was given at the Peers' Club, Tokio, to do posthumous homage to the common friend or perchance benefactor of the gathering and whose memory was so dear to them. It was attended by a large number of persons prominent in the scholarly life

of Tokio, among whom were Baron Makino, the Minister of Education at the time, Prince Iwakura, Marquis Kido, Viscounts Fukuoka and Kano, Barons Hamao, Kikuchi, Kuki, Nomura, Megata, Oki, Kanda, Takahashi, and Kubota, Profs. Yamakawa, Tanakadate, Fujisawa, Miyake, and Watase, Ambassador Tsudzuki, Mr. Hideo Takamine, Mr. Shinkichi Takahashi, Mr. Sen Tsuda, Mr. Taizo Abe, Mr. Shinji Tsuji, and 45 others, the foreign elements being represented by Miss Crosby, Dr. E. Baelz, Mrs. Griffin, Rev. and Mrs. E. R. Miller, Rev. and Mrs. D. Thompson, Mr., Mrs. and Miss Williamson, Rev. MacNair, and Mr. Ballagh.

Baron D. Kikuchi, on the occasion of his visit to America to give lectures at the New York Civic Forum, was requested to pay a tribute from Japan to the memory of Dr. Murray. In February 1910, the Baron, in the capacity of the President of the Imperial Japanese Academy, as well as acting for the Japanese Minister of Education, visited the grave of Dr. Murray, in Elmwood Cemetery, New Brunswick, and placed upon it a wreath. The resolutions naming the Baron to pay suitable respects to Dr. Murray follow:

IMPERIAL ACADEMY RESOLUTIONS.

At the general meeting of the Teikoku Gakushi In (Imperial Academy), held on the 12th of December, 1909, it was unanimously resolved that, as Baron Dairoku Kikuchi, a member of this Academy and holding the office of president of the same, is about to proceed to America, he shall be instructed to visit, as the representative of this Academy, the grave of the late Dr. David Murray, some time superintendent of education in the Department of Education, at whose representation this Academy was established, there to pay proper respects to him.

DEPARTMENT OF EDUCATION'S RESPECTS.

Many eminent services rendered to our country by Dr.

D. Murray, who was invited to Japan as superintendent of education, at the time when the basis of our educational system was not yet firmly established, are well known, and as I [Minister of Education] understand that you are going to visit his grave, representing the Imperial Academy, I request you to act on that occasion also as the representative of the Department of Education, and to pay due respects to him.

A LETTER FROM M. M. SCOTT TO R. FUJISAWA

["A Short History of the Tokio Higher Normal School" published in October 1911 on the occasion of the commemoration of the School's forty years' existence, gives a portrait of Mr. Scott, but does not give any further account than that he was the first teacher engaged and Mr. Gendō Tsuboi acted as his interpreter(p. 7). I thought it a great pity that, so far as I am aware of, nowhere is to be found any published account of one who has done so much for our normal school education. Through the intermediary of my friend Mr. Akai of the Yokohama Specie Bank who lives now in Honolulu, I asked him to furnish me with some particulars of his life, so that I may be able to insert in this appendix a brief sketch of his; but he was so modest as to supply me only with the letter which follows: R. F.]

Mckinley High School,
Honolulu, Hawai.

My dear Doctor
and former pupil:

It seems like shaking hands, in the spirit, over a generation of men in time to address you. To use the dearest and

most expressive phrase in the soft tongue of the gentle Hawaiian, Aloha Oe Nui.

I understand from my friend and former pupil, Mr. Akai, that you wish a few words as to my work in Japan, during my ten years of service in your interesting and beautiful country, and among your delightful people.

I had been much interested in Japan from boyhood; and when an opportunity offered, unexpected and unsought, to go to Japan, I unhesitatingly closed with the offer.

I was engaged for three years, by the Japanese Government, through the auspices of the late Arinori Mori, then Minister Plenipotentiary of Japan at Washington, to proceed to Japan and to teach in the Kaiseijo. I arrived in Japan in September, 1871. After teaching in the Kaiseijo for about eight or ten months, I was invited by the Mombusho to assist in the organization and establishment of a Normal School on modern lines for the education and training of teachers, with the view of introducing a system of public elementary schools, which the Japanese Government then had under consideration. The beautiful and classic site of Seido was selected, where, long before, an ancient seat of learning had been established.

Of the difficulties encountered in thus breaking of new ground, and sowing a small seed, which, I am told, has, thanks to the genius of the Japanese, grown into a large and flourishing tree, I need not speak. The facts of growth are necessarily much better known to you than to me. One thing in this connection, however, now in my advancing age, is very gratifying to look back upon, and that is that, however faltering and imperfect the beginning was in assisting the Japanese to initiate this great work, it was essentially a pioneer experiment in its line. After four years' work in the foregoing line, I was head instructor in the preparatory department of the Tokio University for over three years. I left Japan, with many regrets, in the year 1881.

Although not exactly pertinent the subject in hand, I cannot but express my deepest appreciation and gratitude to all those Japanese gentlemen with whom I have come into contact officially. They have not only treated me justly but generously,....and with an urbanity and delicatec ourtesy nowhere else to be found.

And to the many Japanese students I have taught, for their courtesy, gratitude, devotion to the noble pursuit of truth, and for their self renunciation, I wish to express my unstinted admiration.

M. M. Scott.

Dr. R. Fujisawa.
Honolulu, April 8, 1912.

Addendum——Dr. Inazō Nitobe's "The Intercourse between the United States and Japan" (Johns Hopkins University Studies in Historical and Political Science, Extra Vol. VIII, Baltimore, 1891) gives the following reference to M.M. Scott on page 120.

"The Normal school, opened in 1872 and followed two years later by one for women, has been the work of a Kentucky gentleman, M.M. Scott, M.A., now the Head Master of the largest school in Honolulu. An excellent teacher himself, he introduced the normal school system, which does credit to its framer as well as to his country; for, while the university, planned at first after the American and English models, is now greatly modified by Prussian influences, and while the elementary educational regulations, formerly adopted from the United States, are now more German and Belgian in their character, the normal school system alone may still be said to be essentially American in character.

[The End]

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